

## Math 267 : HW 1

All questions are due **Monday January 14th in Class**

1. Simplify each number to the form  $a + ib$

(a)  $\frac{1}{3-i}$

(b)  $\frac{4+i}{2-3i}$

(c)  $\frac{(12+5i)^2}{2-4i}$

2. Find **all** complex numbers  $z$  with  $z^6 = 27i$ .
3. Use Euler's theorem to rewrite  $\sum_{k=-\infty}^{+\infty} \frac{\sin(kx)}{k^2+1}$  as a sum of complex exponentials.
4. [Review of ODE: eigenvalue problem] Find **all** numbers  $\sigma$  such that,

$$\begin{cases} -\sigma X(x) = X''(x) \\ X(0) = 0 \\ X'(1) = 0 \end{cases}$$

has a solutions  $X(x) \neq 0$ .

5. [Separation of variables] Substitute  $u(x, t) = X(x)T(t)$  into,

$$\begin{cases} \partial_t u = \partial_x^2 u - \partial_x u \\ u(0, t) = u(5, t) = 0 \end{cases}$$

Find an ODE for  $X(x)$ , and another for  $T(t)$ . Do not solve the ODEs.

*Note: you should have boundary conditions for  $X(x)$  and the initial condition for  $T(t)$ .*

6. Fully solve:

$$\begin{cases} \partial_t^2 u = 16\partial_x^2 u \\ u(0, t) = u(\pi, t) = 0 \\ u(x, 0) = 0 \\ \partial_t u(x, 0) = \cos(x) \sin(x) \end{cases}$$

*Use the general solution formula from lecture.*