MATH 121:201 Honours Integral Calculus

Written Home Work (WHW) assignment 1

Due: **Jan. 16, (Fri), 2015, by 2pm.**

Material: [Adams& Essex] Sections 5.1-5.4 and Appendix IV.

In the following problems, you can use the definitions and results contained in the textbook 5.1-5.4, Appendix IV (but, not the exercises).

Show all your work! You have to give clear justifications.

Many of these problems are more difficult than webwork problems or most exercise problems in the textbook.

More challenging problems are starred (*).

Problem 1: Evaluate the following sum:

$$\sum_{k=1}^{2015}(-1)^k k^3$$

Problem 2: Express the given limit as a definite integral. Read the problem carefully.

A.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{n^2}{n^3+i^3}$$

(Your mark will be based on the final expression.)

B.

$$\lim_{n\to\infty}\sum_{k=1}^{2n}\frac{ke^{k^2/n^2}}{n^2}$$

(Your mark will be based on the final expression.)

C. In this problem, you have to clearly justify your answer.

$$\lim_{n\to\infty}\sum_{k=1}^{2n+100}\frac{k\sin(k/n)}{n^2}$$

D. In this problem, you have to clearly justify your answer.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\cos(k/n)\sin(k/n^2)$$

Problem 3: First, recall the definition of Riemann integrability: A function f on the interval $[a,b]$ is said to be Riemann integrable, if for each $\epsilon > 0$, there exists a partition P of $[a,b]$ such that

$$U(f,P) - L(f,P) \leq \epsilon.$$

Use this definition to check rigorously whether the following functions on the interval $[0,1]$ are Riemann integrable.

A.

$$f_A(x) = \begin{cases} \sin\frac{\pi}{x^2} & 0 < x \leq 1, \\ 0 & \text{for } x = 0. \end{cases}$$

B.* Consider a countable set S of real numbers in the interval $[0,1]$, in the form $S = \{r_1, r_2, r_3, ..., r_k, ...\}$. (Note it is assumed that $r_i \neq r_j$ for $i \neq j$.) Let

$$f_B(x) = \begin{cases} 2^{-k} & \text{if } x \in S \text{ and } x = r_k, \\ 0 & \text{if } x \notin S\ . \end{cases}$$