Lec 33.

- Solutions to midterm 2 problems.
- Sequences & series. Ch. 9.

• Sequences.

· $\{a_n\}_{n=1}^{\infty} = a_1, a_2, a_3, a_4 \dots$

does not have to be 1.

e.g. · $a_n = n$. $1, 2, 3, 4, \dots$

· $a_n = 2^n$, $2, 2^2, 2^3, \dots$

· a random sequence

e.g. $a_n = \begin{cases} 1 & \text{if coin flip turns head} \\ 0 & \text{if " " " tail.} \end{cases}$

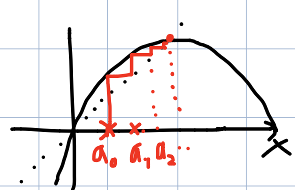
• sequence obtained by some iterations

e.g. $a_1 = 1$, $a_2 = 1$, $a_3 = 1+1 = 3$, $a_4 = a_2 + a_3 = 4$,

$\dots$ $a_n = a_{n-1} + a_{n-2}$ "Fibonacci sequence"

e.g. $F(x) = 2x(1-x)$

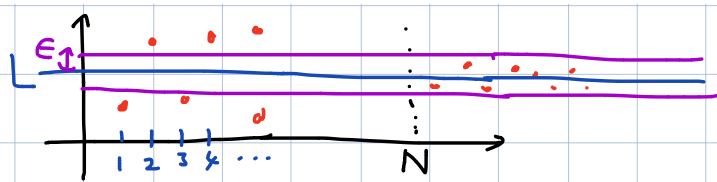
$a_0 = \frac{1}{4}$, $a_1 = F(a_0)$, $\dots$ $a_n = F(a_{n-1})$



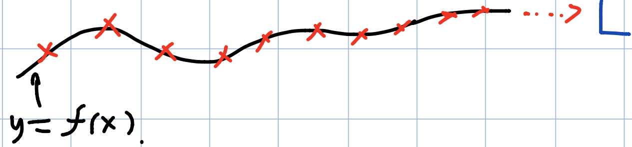
This picture is one of the starting points of discrete dynamical systems ("study of iterations")

Def $\{a_n\}$ converges to the limit L

$\lim_{n\to\infty} a_n = L \iff \forall \epsilon > 0, \exists N$ such that $\forall n \geq N$ $|a_n - L| < \epsilon$.



Rmk For $a_n = f(n)$, if $\lim_{x\to\infty} f(x) = L$ then $\lim_{n\to\infty} a_n = L$



e.g. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ exists? Value?

<sol>.

$$f(x) = \left(1+\frac{1}{x}\right)^x$$

$$g(x) = \ln f(x) = x\ln\left(1+\frac{1}{x}\right)$$

$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \lim_{y\to 0+}\frac{\ln(1+y)}{y}$$

$$\underset{\text{L'Hopital}}{=} \lim_{y\to 0+}\frac{1}{1+y} = 1.$$

$$\therefore \lim_{x\to\infty} g(x) = 1$$

$$\therefore \lim_{x\to\infty} f(x) = \lim_{x\to\infty} e^{g(x)} = e^{\lim_{x\to\infty} g(x)} = e^1 = e.$$

Thus. $\lim_{n\to\infty} a_n = \lim_{n\to\infty} f(n) = \lim_{x\to\infty} f(x) = \underline{e}$. ▨