First Name: _____ Last Name: _____

Student-No: _____ Section: _____

Grade:

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Indefinite Integrals

1. [9 marks] Each part is worth 3 marks. Please write your answers in the boxes.

   (a) Calculate the indefinite integral $I = \int x^2 e^{-3x^3}\, dx$.

   Answer:

   (b) Calculate the indefinite integral $I = \int \frac{3x-2}{x^2+6x+8}\, dx$ for $x > 0$.

   Answer:

(c) (A Little Harder): Calculate the indefinite integral $\int x^2 \sin x \, dx$.

Answer:

Definite Integrals

2. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate $I = \int_0^{\pi/8} \tan^5(2x) \sec^2(2x)\, dx$.

Answer:

(b) Calculate $I = \int_1^e x^2 \ln x\, dx$.

Answer:

(c) (A Little Harder): Calculate $I = \int_0^1 x^3\sqrt{1-x^2}\,dx$.

Answer:

Riemann Sum, FTC, and Volumes

3. [12 marks] Each part is worth 4 marks. Please write your answers in the boxes.

(a) Calculate the infinite sum

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2i}{n^2\left(4 + i^2/n^2\right)}$$

by first writing it as a definite integral. Then, **evaluate this integral**.

Answer:

(b) For $x > 0$ define $F(x) = \int_1^x t^{1/2}\, dt$ and $g(x) = \sqrt{F(x^4)}$. Calculate $g'(2)$.

Answer:

(c) Write a definite integral, with specified limits of integration, for the volume obtained by revolving the bounded region between $x = 2(y-2)^2$ and $x = 6-(y-2)^2$ about the vertical line $x = -2$. **Do not evaluate the integral**.

Answer:

4. (a) [2 marks] Plot the finite area enclosed by $y^2 = 2x$ and $y = x - 4$.

(b) [4 marks] Write a definite integral with specific limits of integration that determines this area. **Do not evaluate the integral**.

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5. A solid has as its base the region in the xy-plane between $y = 1 - x^2/49$ and the x-axis. The cross-sections of the solid perpendicular to the x-axis are squares.

   (a) [4 marks] Write a definite integral that determines the volume of the solid.

   (b) [2 marks] **Evaluate the integral** to find the volume of the solid.

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