MATH 523 HW 2

Question 1. We say that $x \in \mathbb{R}^d$ is an extreme point of a convex set S if whenever $x = ty + (1-t)z$, for $y, z \in S$, $0 < t < 1$, then $x = y = z$.

(1) Prove that for every LP problem which has an optimal solution, the set of feasible solutions has an extreme point. (Warning: It is not true that an optimal solution is always an extreme point.)
(2) Is it true that the set of extreme points of a bounded closed set S is always a closed set?

Question 2. There are 5 different jobs, A, B, C, D, E and 7 workers, 1, 2, ..., 7 who can be assigned to the jobs. The following chart shows what jobs can a worker to be assigned to.

Table 1. Possible work assignments

	A	B	C	D	E
1		*			*
2		*			*
3	*			*	
4		*	*	*	*
5	*			*	*
6		*			
7		*	*	*	

Each worker can be assigned to one job only. For jobs A and C up to two-two workers can be assigned and for the other jobs only one. Your task is to find a job assignment which provides job to as many workers as possible. Formulate the problem as a network flow problem and find the optimum (max flow). Prove that your solution is optimal by finding a min cut. (You can use any ad hoc method to find the max flow.)

Question 3. Here is an Integer Linear Programming problem:

$$\begin{aligned} \text{minimize} \quad & \sum_{j=1}^{m} w_j x_j \\ \text{subject to} \quad & \sum_{j: e_i \in S_j} x_j \geq 1, \qquad i = 1, ..., n \\ & x_j \in \{0, 1\}, \quad j = 1, ..., m \end{aligned}$$

Describe a combinatorial optimization question which can be solved by the ILP problem above. Is there a polynomial-time algorithm which solves your question?

Question 4. (Optional) Let us suppose that you are in charge of scheduling the final exams. There are n courses and m possible 3 hour time slots to use. You know the class lists. Your task is to minimize the conflicts, when a student has two or more exams scheduled to the same time. Formulate this problem as an optimization problem.

Note that large integer programming problems can not be solved in reasonable time. Do you have a suggestion for a possible solution which could be used in practice and gives a reasonable result?

Due date: Oct 5 in class