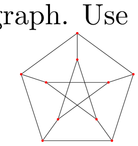
HOMEWORK MATH 441 HW 3

DUE OCT 24, THURSDAY, IN CLASS OR BY EMAIL **BEFORE CLASS** IN A SINGLE, < 1MB PDF FILE

(1) Consider the following problem: *For a given graph, G_n, decide if its edges can be coloured by k colours, such that edges of the same colour are disjoint, they don't share a vertex.* (It is equivalent to say that the edge chromatic number of G_n is at most k) Write an Integer Linear Program (using 0-1 variables) which solves this problem. Explain the program.

(2) Check your ILP program above to find the edge chromatic number of the Petersen graph. Use any ILP solver and include the input and output of the problem.



(3) Suppose we have a 3-SAT formula F. Define the degree of a variable x_i as the total number of times it appears in F (either positively or negatively). Show that the restricted version of 3-SAT in which every variable has degree at most 2 is in P. (Optional: What degree do you think we need for NP-completeness?)

(4) We saw in class how to approximate (max) 3-SAT, by assigning random values to the variables. What is the best approximation you can give to 4-SAT? Explain.

(5) What is the complexity of solving the $n \times n \times n$ Rubik's cube? Do some online research and summarize the results. Give a reference list of the relevant results.

(6) ** *This problem is optional, it is for extra HW credit* (left from previous hw.) Given k subsets of $\{1, 2, \ldots, n\}$ denoted by $S_1, S_2, \ldots, S_k$. The sets satisfy

$$\sum_{i=1}^{k} |S_i| = n.$$

A *shift* by a_i of $S_i = \{s_1, s_2, \ldots, s_\ell\}$ is the set $a_i + S_i = \{s_1 + a_i, s_2 + a_i, \ldots, s_\ell + a_i\}$. The problem is to decide if there are integers a_i such that

$$\cup_{i=1}^{k} \{a_i + S_i\} = \{1, 2, \ldots, n\}$$

Is this problem in P, NP, or something else?