HOMEWORK MATH 441 HW 2

DUE OCT 10, THURSDAY, IN CLASS OR BY EMAIL BEFORE CLASS **IN A SINGLE PDF FILE**

(1) Consider the following problem: *For a given 3-SAT formula decide if it has at least two satisfying assignments.* Is this problem NP-complete? If yes, then prove it. If not, then what is the complexity of this problem?

(2) Consider the following problem: *For a given 3-SAT formula decide if it has at most one satisfying assignments.* Is this problem NP-complete? If yes, then prove it. If not, then what is the complexity of this problem?

(3) Is it possible to find a polynomial reduction of an NP-complete problem to a co-NP-complete problem? Justify your answer.

(4) Give a special case of your selected NP complete problem which can be solved in polynomial time. This time your problem should satisfy the following necessary conditions
   - The input size could be arbitrary large
   - The outcome varies depending on the input (i.e. it is not always YES (or always NO))

   By now I expect you to know your selected problem and related results quite well. Your special case (which can be solved in polynomial time) should cover some **interesting** sub-cases. In particular, it should not be trivial (easy to see) that the problem can be solved in polynomial time. Also list papers (if you've find any) dealing with such examples. Try to construct problems different from the known examples.

   This is an important part of your project. It will be marked separately. Do your best!

(5) ** *This problem is optional, it is for extra HW credit*
   Given k subsets of $\{1, 2, \ldots, n\}$ denoted by $S_1, S_2, \ldots, S_k$. The sets satisfy

$$\sum_{i=1}^{k} |S_i| = n.$$

   A *shift* by a_i of $S_i = \{s_1, s_2, \ldots, s_\ell\}$ is the set $a_i + S_i = \{s_1 + a_i, s_2 + a_i, \ldots, s_\ell + a_i\}$. The problem is to decide if there are integers a_i such that

$$\cup_{i=1}^{k} \{a_i + S_i\} = \{1, 2, \ldots, n\}$$

   Is this problem in P, NP, or something else?