MATH 223 HW 3.

Question 1. A linear transformation, $T : \mathbb{R}^3 \to \mathbb{R}^3$, is defined as $T([1,1,1]^t) = [0,2,-1]^t$, $T([0,2,-1]^t) = [1,1,1]^t$, and $T([2,1,2]^t) = [1,-2,-1]^t$.

a, Find the transformation matrix.

b, Are there vectors, $\vec{u}$, such that $T(\vec{u})$ is orthogonal to $\vec{u}$?

c, Are there vectors, $\vec{u}$, such that $T(\vec{u})$ is parallel to $\vec{u}$?

Question 2. A transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined as

$$T(\vec{u}) = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix} \vec{u}.$$

The eigenvalues of its matrix are $3, 0$, and -4. Find the corresponding eigenvectors.

Question 3. A transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2$, is defined as follows. First we reflect every point over the line $\ell : 2x_1 - x_2 = 0$, and then we rotate the points about the origin counterclockwise with angle $\pi/3$. Find all non-zero vectors which remain the same after the transformation, $T(\vec{u}) = \vec{u}$.

Question 4. Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 1 & -2 \\ -2 & 0 \end{bmatrix}.$$

Question 4. Is there a value, δ, such that the two eigenvalues of the matrix below are 1 and -1?

$$\begin{bmatrix} \delta & \delta^3 \\ \delta^2 & -\delta \end{bmatrix}.$$