Correction to: Upper bounds for the resonance counting function of Schrödinger operators in odd dimensions

Richard Froese

Department of Mathematics
University of British Columbia
Vancouver, British Columbia, Canada

The proof of Lemma 3.4 relies on the incorrect equality $\mu_j(AB) = \mu_j(BA)$ for singular values (for a counterexample, see [S], p. 4.) Thus, Theorem 3.1 as stated has not been proven. However, with minor changes, we can obtain a bound for the counting function in terms of the growth of the Fourier transform of $|V|$. The author thanks Barry Simon for pointing out this error.

Here is the corrected version of Theorem 3.1.

Theorem *Suppose that V is a super-exponentially decaying potential with*

$$\widehat{|V|}(z) \le Ce^{\Phi(|z|)}$$

for a positive, increasing function Φ. Then

$$n(r) \le C\Phi^n(cr) + O(\Phi^{n-1}(cr))$$

for some constants c and C.

These are the changes needed to prove the bound for $|\phi(k)|$ in Lemma 3.4. Using $\det(1+AB) = \det(1+BA)$ and Fan's inequality $\mu_{n+m+1}(AB) \le \mu_{n+1}(A)\mu_{m+1}(B)$ (see [S]) we arrive at

$$\mu_j(T(k)) \le C|k|^{n-2}\mu_{[(j+1)/2]}(F_V^T(-k))\mu_{[(j+1)/2]}(F_{|V|}(-k)) \qquad ()$$

where $[\cdot]$ denotes the integer part. Now

$$\mu_{[(j+1)/2]}(F_{|V|}(-k)) = \left(\mu_{[(j+1)/2]}\mathbf{V}_k)\right)^{1/2}$$

where this time $\mathbf{V}_k$ is the integral operator with integral kernel $\widehat{|V|}(\overline{k}\omega - k\omega')$. We then obtain the bound

$$\mu_{[(j+1)/2]}(F_{|V|}(-k)) \le Ce^{(\Phi-\delta[(j+1)/2]^{(1/(n-1)})/2}$$

and the same bound for $\mu_{[(j+1)/2]}(F_V^T(-k))$. This leads to

$$\mu_j(T(k)) \le Ce^{\Phi-\delta' j^{1/(n-1)}}$$

where $\Phi = \Phi((2+\epsilon)|k|)$ for some $\epsilon > 0$ and $\delta' = \delta 2^{-1/(n-1)}$. The rest of the proof is identical.