1 Math 405/607 E: Ass. 0: Due 23 Sept 2009

1. **toeplitz:** Use the MATLAB function toeplitz to build the matrix A in the demo programs we constructed in class, i.e., which is equivalent to:

   A=diag(-2*ones(N-2,1),0)+diag(ones(N-3,1),1)+diag(ones(N-3,1),-1);

   For $N = 16$ plot the eigenvector corresponding to the eigenvalue with

   the smallest absolute value.

2. **Simple boundary value problem function:** Alter the program demo2.m to construct a function which determines the second order finite difference solution to the boundary value problem

   $$u'' = f(x), \quad u(0) = 0, \quad u(1) = 1$$

   where $f(x)$ is an input function that is supplied by the user. As an example for presenting your results use the function $f(x) = \sin(\pi x)$ and $N = 8$. Compare your result to the exact solution by plotting $u(x_k)$ and $u_{exact}(x_k)$ vs x on one graph and plot the error $|u(x_k) - u_{exact}(x_k)|$ vs x on a separate graph.

3. **Newton's Method:**Write a function newton(fdf,x0,tol) to implement Newton's method for finding a root of a scalar function:

   $$x_{n+1} = x_n - f(x_n)/f'(x_n)$$

   The first input is a handle to a function that computes $f$and f' and $x0$ is the initial guess for the root. Your function should have a loop (either a for ... end or a while ... end - use MATLAB help to discover the syntax for these two operations) to iterate till either $|f(x_{n+1})| < tol$ or $|x_{n+1} - x_n| < tol$. You might want to have a safety valve to avoid an infinite loop. As an example use the function $f = x^5 - 5$ to determine the fifth root of 5. Insert a statement in your function to plot $\log |f(x_n)|$ against n - this curve is a characteristic of Newton's method even in multiple dimensions and can be used to determine if your Newton scheme is converging.