Behaviour of solutions

Which of following statements best capture the large-time behaviour of the solutions of the system

$$\mathbf{x}' = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{bmatrix} \mathbf{x}?$$

A. The solutions are unbounded but not oscillating.

B. The solutions always oscillate between fixed bounds.

C. Each solution approaches a constant value as $t \to \infty$.

D. A solution is always unbounded, but may or may not be oscillating.

Write down the general solution of the system of ODE-s on the previous page.

Answer: The characteristic roots are $1, 1 \pm 2i$. *The solution is*

$$\mathbf{x} = c_1 e^t \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix} + c_2 e^t \begin{pmatrix} 0 \\ \cos 2t \\ \sin 2t \end{pmatrix} + c_3 e^t \begin{pmatrix} 0 \\ \sin 2t \\ -\cos 2t \end{pmatrix}.$$

Repeated eigenvalues

The system $\mathbf{x}' = \mathbf{A}\mathbf{x}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -3 & 2 & 4 \end{bmatrix}$$

has

A. one eigenvalue with three linearly independent eigenvectors

B. one eigenvalue with two linearly independent eigenvectors

C. one eigenvalue with one linearly independent eigenvectors

D. one eigenvalue with no eigenvectors

E. two distinct eigenvalues.

Find the eigenvalue(s) and eigenvector(s) of the previous system, and use it to find a solution to the system, namely

$$\mathbf{x} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t}.$$

How would you find the remaining solutions?