Review Questions

Question 1

Find the particular solution of the differential equation

$$\frac{dy}{dx} + ycos(x) = 4cos(x)$$

satisfying the initial condition $y(0) = 6$.

Question 2

Solve the initial-value problem

$$\frac{dy}{dx} = e^{4x} - 5y$$

satisfying the initial condition $y(0) = 3$

Question 3

Solve the initial value problem

$$y'' + 10y' + 25y = 0$$

given $y(1) = 0; y'(1) = 1$

Question 4

Find the solution to the linear system of differential equations

$$x' = -10x - 12y$$
$$y' = 9x + 11y$$

satisfying the initial conditions $x(0) = 11$ and $y(0) = -9$

Question 5

Newton's Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings. Suppose t is time, T is the temperature of the object, and T_s is the surrounding temperature. The following differential equation describes Newton's Law

$$\frac{dT}{dt} = k(T - T_s)$$

where k is a constant. Suppose that we consider a 95 degree C cup of coffee in a 18 degrees C room. Suppose it is known that the coffee cools at a rate

of 2 degrees C/min when it is 70 degrees C. Answer the following questions.
1. Find the constant k in the differential equation
2. What is the limiting value of the temperature?
3. Use Euler's method with step size h=2 minutes to estimate the temperature of the coffee after 10 minutes.

Question 6 Find the solution to the given initial value problem

$$y'' + 2y' + 2y = \cos t + \delta(t - \frac{\pi}{2}), \qquad y(0) = y'(0) = 0.$$

Question 7 Transform the equation $u^{(4)} - u = \sin t$ into a system of first-order equations if possible. If not, explain why not.

Question 8 Find three linearly independent eigenvectors of the system $\mathbf{x}' = A\mathbf{x}$, where

$$A = \begin{pmatrix} 5 & -3 & -2 \\ 8 & -5 & -4 \\ -4 & 3 & 3 \end{pmatrix}.$$

Question 9 Consider the system

$$\frac{dx}{dt} = x(a - \sigma x - \alpha y), \qquad \frac{dy}{dt} = y(-c + \gamma x),$$

where a, σ, α, c and γ are positive constants.

(a) Find all critical points of the given system. Assume that $a/\sigma > c/\gamma$. Why is this assumption necessary?

(b) Determine the nature and stability characteristics of each critical point.

(c) Show that there is a value of σ between zero and $a\gamma/c$ where the critical point in the interior of the first quadrant changes from a spiral point to a node.

(d) Describe the effect on the two populations as σ increases from zero to $\frac{a\gamma}{c}$.