MATH 215/255 QUIZ 2: SOLUTIONS (SUMMER 2015, T1)

QUESTION 1

(a) Solve ODE for K & F(t)

Set up differential equation

$$2x'' + Kx = F$$

Find

$$x'' = -9c_1cos(3t) - 9c_2sin(3t) = e^t$$

Plug into differential equation

$$(K-18)[c_1cos(3t) + c_2sin(3t)] + (2+K)e^t = F(t)$$

From equation of motion for undamped systems we know:

$$\omega = \sqrt{\frac{K}{m}}$$

$$3 = \sqrt{\frac{K}{2}}$$

Therefore,

$$K = 18$$

Plug K=18 in the above Differential equation to get:

$$F = 20e^t$$

(b) Find $\mathcal{L}[cos^2(\omega t)]$

Trigonometric Identitiy

$$cos^2(\omega t) = \frac{1}{2} + \frac{cos(2\omega t)}{2}$$

Then

$$\mathcal{L}[cos^2(\omega t)] = \mathcal{L}[\frac{1}{2}] + \mathcal{L}[\frac{cos(2\omega t)}{2}]$$

Using the laplace transform tables we get

$$\mathcal{L}[cos^2(\omega t)] = \frac{1}{2s} + \frac{s}{2s^2 + 8\omega^2}$$

QUESTION 2

(i) Find solution to the homogenous equation

$$t^2y'' - 2y = 0$$

Guess:

$$y(t) = t^r$$

$$y'(t) = rt^{r-1}$$

$$y''(t) = r(r-1)t^{r-2}$$

Plug values of y(t) and y"(t) into the above equation:

$$t^r(r^2 - r - 2) = 0$$

Solve for "r":

$$r_1 = -1, r_2 = 2$$

Therefore, the solution to the homogenous equation is:

$$y(t) = c_1\frac{1}{t} + c_2t^2$$

(ii) Use variation of parameters to find a particular solution that solves

$$y'' - \frac{2}{t^2}y = 3 - \frac{1}{t^2}$$

Assume

$$y_p(t) = u_1(t)y_1(t) + u_2(t)y_2(t)$$

where

$$y_1(t) = t^2, y_2(t) = \frac{1}{t}$$

Find the wronskian:

$$W(t) = \begin{vmatrix} t^2 & \frac{1}{t} \\ 2t & \frac{-1}{t^2} \end{vmatrix}$$

$$W(t) = -3$$

Find $u_1(t)$ & $u_2(t)$ using the equation:

$$u_1(t) = -\int \frac{y_2(t)g(t)}{W(t)}dt$$

$$u_2(t) = \int \frac{y_1(t)g(t)}{W(t)}dt$$

where $W(t) = -3$ & $g(t) = 3 - \frac{1}{t^2}$

After plugging in the values for W(t) and g(t) then integrating, we get:

$$u_1(t) = ln(t) + \frac{1}{6t^2}$$

$$u_2(t) = \frac{-t^3}{3} + \frac{t}{3}$$

Using the assumed form of:

$$y_p(t) = u_1(t)y_1(t) + u_2(t)y_2(t)$$

we find that:

$$y_p(t) = t^2 ln(t) - \frac{t^2}{3} + \frac{1}{2}$$

Combining the homogenous and particular solution, we get

$$y(t) = c_1\frac{1}{t} + c_2 t^2 + t^2 ln(t) + \frac{1}{2}$$