The Heaviside function

Express

$$f(t) = \begin{cases} t & \text{if } 0 \le t < 1 \\ t-1 & \text{if } 1 \le t < 2 \\ t-2 & \text{if } 2 \le t < 3 \\ 0 & \text{if } t \ge 3. \end{cases}$$

in terms of the Heaviside function.

A. $f(t) = -u_1(t) - u_2(t) - u_3(t)(t-2)$.

B. $f(t) = t - u_1(t) - u_2(t) - u_3(t)$.

C. $f(t) = t - u_1(t) - u_2(t) - u_3(t)(t-2)$.

D. $f(t) = t - u_1(t) + u_2(t) - u_3(t)(t-2)$.

E. $f(t) = t + u_1(t) - u_2(t) - u_3(t)(t-2)$.

Resonance in a vibrating system

Consider the vibrating system described by the initial value problem

$$ku'' + 9ku = \sin(\omega t), \qquad u(0) = 1, \quad u'(0) = 1.$$

Does there exist a value of k and ω for which the solution will become unbounded as $t \to \infty$?

A. No such k exists, $\omega = 3$

B. No such k or ω exist

C. ω could be arbitrary, $k = 1$

D. k could be any nonzero constant, $\omega = 3$

E. $\omega = 3k$

Second order ODE with variable coefficients

Find the general solution of the equation

$$t^2 y'' - t(t+2)y' + (t+2)y = 2t^3.$$

Answer: $y(t) = C_1 t + C_2 t e^t - 2t^2$

Impulse Functions

Find the solution of the initial value problem

$$y" + 2y' + 2y = \delta(t - \pi), \qquad y(0) = 1, \quad y'(0) = 0.$$

Answer: $y = e^{-t} \cos t + e^{-t} \sin t + u_\pi(t) e^{-(t-\pi)} \sin(t - \pi).$