1. Consider the convex set

$$
K=\{(x, y):|x+y| \leq 1,|x-y| \leq 1\} \subseteq \mathbb{R}^{2}
$$

Find $p_{K}(x, y)$, the Minkowski functional of $K$.
Solution. We observe that $K$ is the closed unit ball in the $\ell^{1}$ norm, i.e.,

$$
K=\left\{(x, y):\|(x, y)\|_{1} \leq 1\right\}, \quad \text { where }
$$

$$
\|(x, y)\|_{1}=\max (|x+y|,|x-y|)=|x|+|y| .
$$

By a result discussed in class,

$$
p_{K}(x, y)=\|(x, y)\|_{1}=|x|+|y| .
$$

