Math 121 Final Exam - Sample 2
Total time allowed: 3 hours

1. (10 points) Determine whether the following series converges absolutely, converges conditionally, or diverges:
$$\sum \frac{\sin(n+1/2)\pi}{\ln\ln n}$$
2. (10 points) Find the specified Taylor series representation of the following, and determine where the series representation is valid.
$$f(x) = x\ln(x) \text{ in powers of } x-1.$$
3. (20 points) Find the Fourier series sine series for $f(t) = \pi - t$ on $[0, \pi]$.
4. (15 points) Sketch and find the area of the region R bounded by the coordinate axes and the parabolic arc
$$x = \sin^4 t, y = \cos^4 t.$$
5. (20 points) Calculate $Pr(|X-\mu| \geq 2\sigma)$ for the exponential distribution with density $f(x) = ke^{-kx}$ on $[0, \infty)$.
6. (15 points) Find the centre of mass of a semicircular plate occupying the region $x^2 + y^2 \leq a^2, y \geq 0$, if the density at distance s from the origin is ks g/cm^2.
7. (15 points) Find the length of the following curve from $(-1, 1)$ to $(0, 1+\sqrt{2/3})$.
$$2(x+1)^3 = 3(y-1)^2$$
8. (15 points) Find the volume of the solid obtained by rotating the region bounded by the following about the x-axis:
$$y = \frac{1}{1+x^2},\ y = 2,\ x = 0, \text{ and } x = 1.$$
9. (10 points) Evaluate the following integral:
$$\int \frac{\ln(\ln x)}{x}\,dx$$
10. (5+5+5 points) Determine of the following statements are True or False. If true, prove it. Else give a counterexample to show falsehood.
    (a) If $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty}(-1)^n a_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges absolutely.
    (b) If $\sum a_n$ and $\sum b_n$ both diverge, then so does $\sum(a_n + b_n)$.
    (c) If neither $\{a_n\}$ not $\{b_n\}$ converges, then $\{a_n b_n\}$ does not converge.
11. (5 points) Test the following series for convergence:
$$\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}$$