Math 121 Practice Problem Set 3
(Based on Chapters 5 and 6)

1. Evaluate the sum $\sum_{j=1}^{n} \frac{2j+1}{j^2(j+1)^2}$.

   (Answer: $\frac{n(n+2)}{(n+1)^2}$)

2. Find the derivative of the function $g(\theta) = \int_{e^{\sin\theta}}^{e^{\cos\theta}} \ln x \, dx$.

   (Answer: $-\sin\theta\cos\theta(e^{\cos\theta} + e^{\sin\theta})$)

3. Find the upper and lower Riemann sums $U(f, P_n)$ and $L(f, P_n)$ for $f(x) = \frac{1}{x}$ on the interval $[1, 2]$ with $P_n = \{2^{i/n} : 0 \le i \le n\}$. Then compute the limits of these Riemann sums as $n \to \infty$ and verify that they both converge to the value of the integral.

(Answer: $U(f, P_n) = n(2^{1/n} - 1)$, $L(f, P_n) = \frac{U(f,P_n)}{2^{1/n}}$, limit$= \ln 2$)

4. Evaluate the integral $\int \frac{x\,dx}{4x^4 + 4x^2 + 5}$.

(Answer: $\frac{1}{8}\tan^{-1}(x^2 + \frac{1}{2}) + C$)