Math 121 Assignment 7

Due Friday March 11

1. For each of the following sequences, evaluate the limit, if it exists:

$$(a) a_n = \frac{n^2 - 2\sqrt{n} + 1}{1 - n - 3n^2}, \quad (b) a_n = n - \sqrt{n^2 - 4n}, \quad (c) a_n = \frac{(n!)^2}{2n!}.$$

2. Determine whether the limit of the sequence

$$a_1 = 3, a_{n+1} = \sqrt{15 + 2a_n}$$

exists, and if so, find the limit.

3. Find the sums of the series

$$(a) \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \cdots,$$

$$(b) \frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \cdots.$$

4. When dropped, an elastic ball bounces back up to a height three quarters of that from which it fell. If the ball is dropped from a height of 2 meters, and is allowed to bounce up and down indefinitely, what is the total distance it travels before coming to rest?

5. Decide whether the following statements are true or false. If true, prove it. If not, give a counterexample.
   (a) If $\sum a_n$ diverges and $\{b_n\}$ is bounded, then $\sum a_n b_n$ diverges.
   (b) If $a_n > 0$ and $\sum a_n$ converges, then $\sum (a_n)^2$ converges.
   (c) If $\sum a_n$ converges, then $\sum (-1)^n a_n$ converges.
   (d) If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum (-1)^n a_n$ converges absolutely.

6. Determine the convergence or otherwise of the following series.

$$(a) \sum_{n=1}^{\infty} \frac{2^{2n}(n!)^2}{(2n)!} \quad (b) \sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2} \quad (c) \sum_{n=10}^{\infty} \frac{1}{n \ln n (\ln \ln n)^2}$$

7. Determine the values of x for which the series below converge absolutely, converge conditionally or diverge.

$$(a) \sum_{n=1}^{\infty} \frac{(2x+3)^n}{n^{1/3} 4^n} \quad (b) \sum_{n=1}^{\infty} \frac{1}{n}\left(1 + \frac{1}{x}\right)^n \quad (c) \sum_{n=1}^{\infty} \frac{(2n)! x^n}{2^{2n}(n!)^2}$$