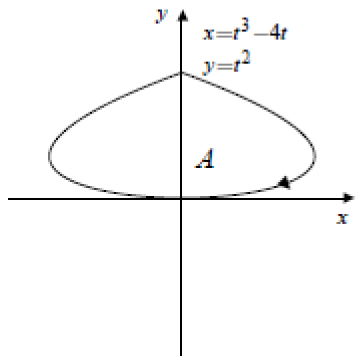

Figure 1: 1

Math 121: Midterm 2 solutions

1. $x = t^3 - 4t$, $y = t^2$, $-2 \leq t \leq 2$.

$$\begin{aligned} Area &= \int_{-2}^{2} t^2(3t^2-4)dt \\ &= 2\int_{0}^{2}(3t^4-4t^2)dt \\ &= 2(3t^5/5-4t^3/3)|_0^2 = \frac{256}{15} sq.units. \end{aligned}$$

2. We have

$$x'(t) = 0.12 - \frac{10x(t)}{1000+2t}.$$

This is a linear first order ODE with initial condition $x(0) = 50$.

3. The series can be written as

$$\sum_{n=1}^{\infty}(-1)^n\frac{n^{100}2^n}{\sqrt{n!}}.$$

$$\begin{aligned} \lim\frac{a_{n+1}}{a_n} &= \frac{(n+1)^{100}2^{n+1}}{\sqrt{(n+1)!}} / \frac{n^{100}2^n}{\sqrt{n!}} \\ &= \lim 2(\frac{n+1}{n})^{100}\frac{1}{\sqrt{n+1}} \\ &= 0. \end{aligned}$$

So the series converges absolutely.

4. Let $a_1 = 1$ and $a_{n+1} = \sqrt{1+2a_n}$ for $n = 1, 2, 3, ...$ Then we have $a_2 = \sqrt{3} > 1$. If $a_{k+1} > a_k$ for some k, then

$$a_{k+2} = \sqrt{1+2a_{k+1}} > \sqrt{1+2a_k} = a_{k+1}.$$

Thus, $\{a_n\}$ is increasing by induction. Let $\lim a_n = a$. Then

$$a = \sqrt{1+2a}.$$

$$a^2 - 2a - 1 = 0.$$

$$a = \frac{2 \pm \sqrt{8}}{2}.$$

Since $a > a_1 = 1$, we have $\lim a_n = \frac{2+\sqrt{8}}{2}$.

5. (a) True. With any $\epsilon > 0$, there exists N, such that when $n > N$, we have $\sqrt{a_n} < \epsilon < 1$. Since we must have $a_n < \sqrt{a_n}$, the series $\sum_n a_n$ is convergent by comparison.

   (b) False.