Math 300 Homework 1 Solution

1. (Section 1.1 Q9)

$$\frac{2+3i}{1+2i} - \frac{8+i}{6-i} = \frac{2+3i}{1+2i} \times \frac{1-2i}{1-2i} - \frac{8+i}{6-i} \times \frac{6+i}{6+i}$$
$$= \frac{8-i}{5} - \frac{47+14i}{37}$$
$$= \frac{61}{185} - \frac{107}{185}i$$

2. (Section 1.1 Q16d)

$$i^{-4321} = i^{4(-1081)+3} = i^3 = -i$$

3. (Section 1.2 Q7g)

$$|z| = 3|z-1|$$
$$\sqrt{x^2+y^2} = 3\sqrt{(x-1)^2+y^2}$$
$$x^2+y^2 = 9(x^2-2x+1+y^2)$$
$$8x^2+8y^2-18x+9 = 0$$
$$8(x-\frac{9}{8})^2 + 8y^2 + 9 - \frac{81}{8} = 0$$
$$(x-\frac{9}{8})^2 + y^2 = \frac{9}{64} = \left(\frac{3}{8}\right)^2$$

All points on the circle of radius $\frac{3}{8}$ with centre at $\left(\frac{9}{8}, 0\right)$.

4. (Section 1.3 Q7h)
Let $z_1 = -\sqrt{7}(1+i) = -\sqrt{7} - \sqrt{7}i$.
$r_1 = \sqrt{7+7} = \sqrt{14}$ and $\theta_1 = 5\pi/4$.
Let $z_2 = \sqrt{3}+i$.
$r_2 = \sqrt{3+1} = \sqrt{4} = 2$ and $\theta_2 = \pi/6$.

$$z = \frac{-\sqrt{7}(1+i)}{\sqrt{3}+i} = \frac{z_1}{z_2}$$
$$= \frac{\sqrt{14}}{2}\text{cis}\left(\frac{5\pi}{4} - \frac{\pi}{6}\right)$$
$$= \frac{\sqrt{14}}{2}\text{cis}\left(\frac{13\pi}{12}\right)$$
$$= \frac{\sqrt{14}}{2}\text{cis}\left(\frac{-11\pi}{12}\right)$$

$$arg(z) = \frac{-11\pi}{12} + 2k\pi$$

5. (Section 1.4 Q3c)

$$(1+i)^6 = \left(\sqrt{2}e^{\frac{\pi i}{4}}\right)^6 = 8e^{\frac{3\pi i}{2}}$$