Homework 1 - Math 321, Spring 2012

Due on Friday January 13

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1. Does the sequence of functions
$$f_n(x) = nxe^{-nx}$$
converge pointwise on $[0, \infty)$? Is the convergence uniform on this interval? If yes, give reasons. If not, determine the intervals (if any) on which the convergence is uniform.
2. Let $\{f_n : n \geq 1\}$ and $\{g_n : n \geq 1\}$ be real-valued functions on a set X, and suppose that both sequences converge uniformly on X. Show that the sequence $\{f_n + g_n : n \geq 1\}$ converges uniformly on X. Give an example showing that $\{f_n g_n : n \geq 1\}$ need not converge uniformly on X.
3. Fix $a, b \in \mathbb{R}$, $a < b$. Let $f_n : [a, b] \to \mathbb{R}$ satisfy $|f_n(x)| \leq 1$ for all x and n. Show that there is a subsequence $\{f_{n_k}\}$ such that $\lim_{k\to\infty} f_{n_k}(x)$ exists for each rational $x \in [a, b]$.