

Math 105 Course Outline

Week 4

Overview

Over the past few weeks, we have studied vectors and partial derivatives, and how they can be used to optimize functions of two variables. We now return to our attention to functions of one variable, as we start on a new subject, integration, which is a central theme of this course.

This week we will start by trying to measure the area under the graph of a given (non-negative) function. If the graph is just a straight line or a circle, we can use familiar formulas from geometry to find this area. For more complex curves, we will first try to *estimate* the area by splitting it into many small, simple pieces and adding up the areas of each. It will be helpful to have a concise way to express these long sums of many small numbers, so we will learn how to make use of *sigma notation* for this purpose. Finally, we will arrive at the precise area using the concept of *limits* from differential calculus.

This leads to the key concept of the *definite integral* of a function between two points. Although we originally wanted to understand the area underneath a graph, we will see that the definite integral has an intuitive geometric meaning even if the function is negative (and there is no “underneath” to speak of). The definite integral also has applications well beyond the measurement of area, just as the derivative has many uses beyond measuring the slope of a graph.

At first, our ability to actually calculate definite integrals will be rather limited, but we can still study some of the ways that definite integrals behave under various operations like addition, subtraction, and multiplication by constants. Starting next week, we will learn that there is a very close connection between integrals and derivatives, and we will use this relationship to unlock the full power of integration.

Learning Objectives

These learning objectives describe a minimum level of understanding that is examinable. By the end of the week, having participated in lectures, worked through the indicated sections of the textbook and other resources, and done the suggested problems, you should be able to do all of these objectives below independently.

Ref	Learning Objective
04-01	<p>Estimating areas under curves</p> <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Objective 1: Be able to describe a <i>regular partition</i> of an interval, a <i>Riemann sum</i> for a function on a given interval (including the specific cases of <i>left</i>, <i>right</i>, and <i>midpoint Riemann sums</i>), and how they can be used to approximate area. [Conceptual]</p> </div> <p>Example problem: <i>Sketch an approximation for the area under the graph of $f(x) = x^2$ on the interval $[-2, 2]$ with a regular partition into 4 subintervals, using the midpoint of each subinterval.</i></p> <p>Reading: Text §5.1 (pp. 306-310)</p> <p>Practice problems: Text p. 315-316: 3-4, 7-8.</p> <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Objective 2: Be able to compute specific Riemann sums for a function on a given interval. [Procedural]</p> </div> <p>Example problem: <i>Calculate the left Riemann sum for $f(x) = \cos(x)$ on the interval $[0, \frac{\pi}{2}]$ using $n = 4$.</i></p> <p>Reading: Text §5.1 (pp. 309-312)</p> <p>Practice problems: Text p. 316-317: 5-6, 17-26</p>
04-02	<p>Summation and sigma notation</p> <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Objective 1: Be able to read and interpret an expression in sigma notation as the sum of a series of numbers; conversely, be comfortable with using sigma notation to concisely describe a particular sum. [Conceptual]</p> </div> <p>Example problem: <i>Express the sum $5 + 7 + 9 + 11 + \cdots + 23 + 25$ using sigma notation.</i></p> <p>Reading: Text §5.1 (p. 313)</p> <p>Practice problems: Text p. 317: 31-33</p> <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Objective 2: Be able to manipulate expressions written in sigma notation, and to use Theorem 5.1 to evaluate certain sums. [Procedural]</p> </div> <p>Practice problems: Text p. 317: 34(i)</p> <div style="border: 1px solid black; padding: 5px;"> <p>Objective 3: Given a function $f(x)$, be able to express Riemann sums for $f(x)$ on a given interval using sigma notation. Conversely, given a Riemann sum in sigma notation, be able to identify a function and an interval which give rise to that sum. [Conceptual]</p> </div> <p>Reading: Text §5.1 (pp. 313-315)</p> <p>Practice problems: Text p. 318: 49-56</p>

Ref	Learning Objective
04–03	<p>Definite integrals</p> <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Objective 1: Be able to define a definite integral as the limit of Riemann sums; conversely, be able to recognize a given limit of Riemann sums as corresponding to a definite integral. [Recall]</p> </div> <p>Example problem: Express the integral $\int_1^3 \ln(x)dx$ as a limit of Riemann sums.</p> <p>Reading: Text §5.2 (pp. 322-324)</p> <p>Practice problems: Text p. 332: 19-22 (make sure you have achieved Objective 3 from 04-02 first).</p> <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Objective 2: Be able to interpret a definite integral geometrically as the <i>net area</i> (also called <i>signed area</i>) of an associated region R. Conversely, given a sketch of a region, be able to express the area as a definite integral. [Conceptual]</p> </div> <p>Reading: Text §5.2 (pp. 320-322, 326-327)</p> <p>Practice problems: Text p. 332: 31-38</p> <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Objective 3: Be able to compute definite integrals of simple functions by using the geometric interpretation and facts about area. [Procedural]</p> </div> <p>Reading: Text §5.2 (pp. 325-326)</p> <p>Practice problems: Text p. 332: 23-30, 66-69</p> <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Objective 4: Be able to use algebraic properties of integrals to deduce the value of one definite integral from information about related integrals. [Conceptual]</p> </div> <p>Reading: Text §5.2 (pp. 327-330)</p> <p>Practice problems: Text p. 332: 39-44</p> <div style="border: 1px solid black; padding: 5px;"> <p>Objective 5: Be able to compute definite integrals of polynomial functions using the limit definition. [Procedural]</p> </div> <p>Reading: Text §5.2 (pp. 330-331)</p> <p>Practice problems: Text p. 333: 45-50</p>