Math 105 Course Outline

Week 2

Overview

This week, we will be learning about *partial derivatives* of functions of two variables. These are analogs of the derivative of a function of a single variable, and many aspects of these will be familiar. However, there are certain subtleties which must be addressed.

We'll then begin to look at applications of partial derivatives to the study of certain real-world problems.

Finally we will apply partial derivatives to study maxima and minima of functions of two variables. Again, there are similarities to the use of derivatives in studying extrema of functions of a single variable, but we will see that surfaces exhibit a greater diversity of behaviours than simple curves.

Learning Objectives

These should be considered a minimum, rather than a comprehensive, set of objectives.

By the end of the week, having participated in lectures, worked through the indicated sections of the textbook and other resources, and done the suggested problems, you should be able to independently achieve all of the objectives listed below.

Ref	Learning Objective

02–01 Definition and interpretation of partial derivatives

Objective 1: Give the limit definition of the partial derivatives f_x and f_y of a function $f(x, y)$ of two variables at a point (a, b). [Recall]

Example problem: *If $f(x, y) = 2x^2 - 3y^2 - 2$, write the limit definition of the partial derivative f_x.*

Objective 2: Given a function $f(x, y)$ representing some physical or mathematical relationship, describe the meanings of, and give interpretation of, the partial derivatives f_x and f_y in terms of the meanings of f, x, and y. [Conceptual]

Example problem: *The production P of a given factory is described as a function of capital investment K (measured in dollars) and labour L (measured in worker hours.) Give an economic interpretation of the partial derivatives $\frac{\partial P}{\partial K}$) and $\frac{\partial P}{\partial L}$.*

Reading: Text §12.4 (pp. 810 – 813)

Practice problems: Text p. 818: *1 – 6*

02–02 Computation of and with first and higher order partial derivatives

Objective 1: Compute the first and higher partial derivatives of a function of two variables. [Procedural]

Example problem: *Let $f(x, y) = x \sin y$. Find f_x and f_y. Then find all of the second partial derivatives of f.*

Reading: Text §12.4 (pp. 812 – 814)

Practice problems: Text p. 818: *7 – 16*

Objective 2: Recognize that mixed partial derivatives are usually equal, and determine for a given function if they are not. (Clairaut's theorem) [Conceptual]

Example problem: *Given two functions $f(x, y) = 2y$ and $g(x, y) = 3x$, determine whether there exists a function $F(x, y)$ such that $F_x = f$ and $F_y = g$ (Clairaut's theorem) [Conceptual]*

Reading: Text §12.4 (p. 814)

Practice problems: Text p. 818: *17 – 30*

02–03 Applications of partial derivatives

Objective 1: For simple mathematical models, interpret mathematical statements in terms of the situation being modelled, and vice-versa. [Conceptual]

Practice problems: Text p. 820: *56 – 71*

Ref	Learning Objective
02–04	Differentiability

Reading: Text §12.4 (pp. 816 – 818)

Objective 1: Explain in your own words what it means for a function of two variables to be differentiable at a point (a, b). [Conceptual]

Objective 2: Given a function $f(x, y)$, use Theorem 12.5 to decide whether f is differentiable at (a, b).

Practice problems: Text p. 819: p. 821: *72, 73.* For these problems, use Theorem 12.5 to show differentiability. You do not need to find ϵ_1 and ϵ_2.

Notes:
1) You will not need to consider functions of more than two variables.
2) You are not expected to be able to prove the theorems (12.4, 12.5, and 12.6) in this section — only to state, interpret, and apply them. Pay careful attention to the relevant definitions and theorems highlighted in boxes, since they summarize the main aspects of this material. The $\epsilon - \delta$ justification is not part of the syllabus, nor will you be tested on the mathematical rigorous definition of differentiability involving the error functions ϵ_1 and ϵ_2.

For instance, you should be able to interpret the definition of differentiability of a function f as follows:

Suppose $z = f(x, y)$ is differentiable at a point (a, b). If (x, y) changes to $(x + \Delta x, y + \Delta y)$, the resulting change Δz in z is well-approximated by the formula $f_x(a, b)\Delta x + f_y(a, b)\Delta y$.

Ref	Learning Objective
02–05	Maxima and minima for functions of two variables

Reading: Text §12.8 (pp. 853 – 860)

Objective 1: Give the definition of a local minimum (respectively, maximum) of a function $f(x, y)$ of two variables. [Recall]

Objective 2: Interpret the definition of a local minimum (respectively, maximum) of a function $f(x, y)$ by sketching a picture of a local minimum (resp. maximum) and drawing a suitable open disk as called for in the definition. [Conceptual]

Objective 3: State a theorem which gives criteria for a function $f(x, y)$ to have a local minimum (maximum) at a point (a, b), in terms of the value of the partial derivatives f_x and f_y at (a, b). [Conceptual]

Objective 4: Explain how to use the theorem in the previous learning objective to find potential local maxima and minima for a differentiable function $f(x, y)$. [Conceptual]

Objective 5: Explain what is required for a point (a, b) to be a critical point of a function $f(x, y)$. [Conceptual]

Objective 6: Find critical points of a given function $f(x, y)$ of two variables. [Procedural]

Practice problems: Text p. 861: *9 – 14*

Objective 7: Give the definition of a saddle point for a function $f(x, y)$, and explain the significance of such points. [Recall, Conceptual]

Objective 8: Classify critical points of a given function $f(x, y)$ using the second derivative test. [Procedural]

Objective 9: Solve word problems involving the finding of maximum or minimum values. [Procedural; Problem Solving]

Practice problems: Text p. 861: *15 – 28*; *29 – 32*; *33 – 36*;

Notes: 1) The theory and techniques involved in finding absolute maxima and minima (pp. 858 – 860) will be be discussed in Week 3; skip this section of the reading and problems for now.