Math 105 Course Outline

Week 12

Overview

Last week we started on infinite series. In particular we learned about two tests of convergence and/or divergence: namely the divergence test and the integral test. This week we will learn a few more such tests designed to verify convergence or divergence of an inifinite series, even when we cannot evaluate the series exactly. We will then begin the study of a very special class of functions whose functional forms are given by infinite series.

Learning Objectives

These should be considered a minimum, rather than a comprehensive, set of objectives.

By the end of the week, having participated in lectures, worked through the indicated sections of the textbook and other resources, and done the suggested problems, you should be able to independently achieve all of the objectives listed below.

Ref	Learning Objective
12–01	Ratio and comparison tests

Objective 1: State the ratio test, comparison test and limit comparison test for convergence or divergence of an infinite series with positive terms. [Recall/Conceptual]

Objective 2: Use the tests learned so far to check if an infinite series converges or diverges. Be able to recognize forms that indicate the applicability or otherwise of certain tests. [Procedural]

Example problem: *For each of the examples* $\sum_{k=1}^{\infty} 2^k k!/k^k$ *and* $\sum_{k=1}^{\infty} \frac{k^8}{k^{11}+3}$ *does the series converge or diverge? .*

Objective 3: Give the definition of absolute and conditional convergence. Give examples of some absolutely convergent and some conditionally convergent series. [Recall]

Reading: Text §7.2 (pp. 569 – 575, excluding the portion on root test)

Practice problems: Text p. 575–577: *1– 18, 27–65.*

Ref	Learning Objective
12–02	Approximating functions with polynomials

Objective 1: Given a function f that can be differentiated up to any order, describe the nth-order Taylor polynomial for f with center a. [Procedural]

Example problem: *Find the* n*th-order Taylor polynomial centered at* 0 *of the function* $f(x) = (1+x)^{-2}$*, for* $n = 0, 1, 2$.

Objective 2: Given a function f that can be differentiated up to any order, explain why the nth order Taylor polynomial may be considered a good approximation to f. Use Taylor's theorem and Taylor's remainder formula to estimate the error in approximation of f. [Procedural/Conceptual]

Example problem: *Use the Taylor remainder term to estimate the maximum error in the apprximation* $\sin x \approx x - \frac{x^3}{6}$ *on the interval* $[-\frac{\pi}{4}, \frac{\pi}{4}]$.

Reading: Text §9.1 (pp. 589 – 599)

Practice problems: Text §9.1, p. 599-600: *1 – 46, 53–64*

Ref	Learning Objective
12–03	Properties of power series

Objective 1: Define a power series, and the radius of convergence of a power series. Use the radius of convergence to determine an interval where the series converges absolutely. [Recall]

Objective 2: Given a power series, be able to find the radius of convergence of a power series. [Procedural]

Example problem: *Find the radius of convergence of the power series* $\sum_k \frac{(-2)^k(x+3)^k}{3^{k+1}}$. *Use this to find an interval where the series converges absolutely.*

Objective 3: Be comfortable with algebraic manipulations of power series, such as sums, differences, multiplication by a power, composition, integration and differentiation. [Procedural]

Example problem: *Find the power series representation of* $g(x) = \ln(1-3x)$ *using* $f(x) = 1/(1-3x)$.

Reading: Text §9.2 (pp. 602 – 609)

Practice problems: Text §9.2 p. 609: *1– 44, 46–49, 52–57*