Math 105 Course Outline

Week 11 Goals

Overview

In first-semester calculus, you learned what it meant to talk about the limit of a function. We begin this week by discussing what it means to talk about the limit of an infinite list of numbers (which we call an *infinite sequence*). For example, the sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ converges to zero while both of the sequences $0, 1, 2, 3, \ldots$ and $-1, 1, -1, 1, \ldots$ fail to tend to a limit (in other words, *diverge*).

The next question we turn to is: Given an infinite list of numbers, what does it mean to add them all up? (A sum of an infinite list of numbers is called an *infinite series*.) We discuss certain special types of infinite series (such as geometric series, telescoping series, and p-series), as well as tests that allow you to determine convergence and divergence for very general series.

Learning Objectives

These should be considered a minimum, rather than a comprehensive, set of objectives.

By the end of the week, having participated in lectures, worked through the indicated sections of the textbook and other resources, and done the suggested problems, you should be able to independently achieve all of the objectives listed below.

Ref	Learning Objective
11–01	Sequences: Basic properties

Objective 1: Give the definition of a **sequence** $\{a_n\}$. See p. 527. [Conceptual]

Objective 2: Compute the first several terms of a sequence given to you either by an explicit formula or by a recursive definition. [Procedural]

Example problem: If $a_{n+1} = 3a_n - 1$ and $a_1 = 1$, find all of a_1, a_2, a_3, and a_4.

Reading: Text §8.1 (pp. 526 – 528)
Practice problems: Text pp. 534–535: *9–16.*

Objective 3: Explain what it means for a sequence to have a limit (converge) or diverge. Given several terms of a sequence, be able to formulate a guess as to its limit. [Conceptual + Procedural]

You **do not** have to understand the formal definition of the limit of a sequence presented on pp. 545–546.

Reading: Text §8.1 (p. 529)
Practice problems: Text p. 535: *23-30.*

Objective 4: [Procedural] Know how to compute limits of sequences using either of the following methods:

- Writing the nth term as $f(n)$, where f is a function for which $\lim_{x\to\infty} f(x)$ exists. See Theorem 8.1 on p. 537.
- Applying the squeeze theorem. See §8.2 (p. 542).

Example problem: If $a_n = e^n/(1 + 3e^n)$, what is $\lim_{n\to\infty} a_n$?

Example problem: Find the limit of the sequence $a_n = \frac{n^2+n\sin(n)}{n^3+1}$.

Reading: Text §8.2 (pp. 537, 542)
Practice problems: Text p. 546: *9-26, 43–46.*

Ref	Learning Objective
11–02	Sequences: Continued

Objective 1: Apply the properties of limits summarized in Theorem 8.2 on p. 538. [Procedural]

Example problem: If a_n and b_n are two sequences with $\lim_{n\to\infty} a_n = -3$ and $\lim_{n\to\infty} b_n = 10$, find $\lim_{n\to\infty}(3a_n - 7b_n)$.

Practice problems: Text p. 547: *63, 67, 69.*

Objective 2: Recognize when a sequence is **increasing**, **decreasing**, **bounded**, or **monotone**. [Conceptual]

Objective 3: Be able to state Theorem 8.5: A bounded monotonic sequence converges. Recognize (using the skills acquired in Objective 2) when this result applies. [Conceptual]

Example problem: Which of the adjectives **increasing**, **decreasing**, **bounded** apply to the sequence whose nth term is $a_n = (-1)^n/n$? What about $a_n = n^2$? What about $a_n = \frac{n}{n+1}$?

Reading: Text §8.2 (pp. 539–540)
Practice problems: Text p. 546: *1–4.*

Objective 4: Recognize examples of geometric sequences and determine whether they converge or diverge. [Conceptual and Procedural]

Example problem: Does the sequence $\{(0.95)^n\}$ converge or diverge?

Reading: Text §8.2 (pp. 540–541)
Practice problems: Text p. 546: *35–42.*

Ref	Learning Objective
11–03	Infinite Series

Objective 1: Give the definition of an **infinite series** $\sum_{k=1}^{\infty} a_k$ and explain what is meant by the **sequence of partial sums**. Relate the convergence or divergence of the series to the sequence of partial sums. [Conceptual]

Example problem: Compute the first four partial sums of the series $\sum_{n=1}^{\infty}(-1)^n/n$.

Example problem: Find a formula for the nth partial sum of the series $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$. Use this to determine the value of the infinite series.

Reading: Text §8.1 (pp. 532–534, 552–553)
Practice problems: Text pp. 534–535: *6, 7, 8, 60–67* and pp. 553–554: *47–58*.

Objective 2: Recognize when a geometric series converges and be able to compute its sum in that case. [Conceptual + Procedural]

Example problem: Does the series $\sum_{k=1}^{\infty}(-0.95)^k$ converge or diverge? If it converges, what is its value?

Reading: Text §8.3 (pp. 550–552)
Practice problems: Text pp. 553–554: *7–46, 59*.

Objective 3: Be able to manipulate convergent series according to the rules described in Theorem 8.8 on p. 557. [Procedural]

Example problem: Evaluate the infinite series $\sum_{k=1}^{\infty} \frac{3-4^k}{5^k}$.

Reading: Text §8.3 (pp. 557–559)
Practice problems: Text p. 567: *9–14*.

Objective 4: [Procedural] Be able to use both of the following tests:

- The divergence test (Theorem 8.9, p. 559), which gives a sufficient condition for a series to diverge.
- The integral test (Theorem 8.11, p. 561), which shows the equivalence between the convergence of a series and that of an associated integral.

Example problem: Explain why the series $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{\sqrt{k+1}}$ diverges.

Example problem: Does the series $\sum_{k=2}^{\infty} \frac{1}{k \ln k}$ converge or diverge? What about $\sum_{k=1}^{\infty} \frac{1}{\sqrt[5]{k}}$?

Reading: Text §8.4 (pp. 559–564)
Practice problems: Text p. 567: *15–30, 44–49*.