

Math 105 Week 1

January 4-6, 2011

1. LESSON PLAN

In this introductory week, we will define scalar functions of several variables, and discuss their geometric interpretations. This is a subset of the material contained in sections 12.1 and 12.2 of the textbook. A rudimentary knowledge of vectors and vector arithmetic will be useful for this discussion, and will be presented briefly. Topics to be covered include:

- Equation of a plane passing through a point and perpendicular to a vector \mathbf{n} (12.1, p774-775)
- Definition of normal vector to a plane (12.1, p775-776)
- Orthogonal and parallel planes (12.1, p777)
- Trace of a surface (12.1, p778)
- Functions, domain and range with two independent variables (12.2, p789)
- Graphs of functions of two variables (12.2, p790)
- Level curves (12.2, p792-795)

Discussion of “normal vector to a plane” will assume the knowledge of vectors, along with the basic operations of addition and scalar multiplication. The length and direction of a vector should also be defined.

The following topics are *not* in the syllabus: equation of a plane passing through three points (in particular, cross product of two vectors), quadric surfaces, identification of surfaces (such as cylinder, ellipsoid, paraboloid, hyperboloid) from their defining equations, functions of more than two variables.

While you will not be asked to identify a surface in a test, you should use the content of these lectures to develop the skill of sketching at least roughly the shape of a surface given certain standard equations, such as

$$x^2 + y^2 + z^2 = 1, \quad z = \frac{x^2}{a^2} + \frac{y^2}{b^2}, \quad z^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2}.$$

This will be a very useful tool in later sections.

2. LEARNING OBJECTIVES

By the end of the week and after going through the practice problems, you should be able to:

1. distinguish (notationally and conceptually) points and vectors in three-dimensional space,
2. add and subtract vectors,
3. multiply a vector by a number (scalar),
4. identify if two vectors are parallel, orthogonal or neither,
5. find the length and direction of a vector,
6. identify a linear equation in three variables as a plane in three-dimensional space,
7. read off a normal vector to a plane from its defining equation,
8. decide whether a point lies in the plane or not,

9. check if two planes are parallel, orthogonal or neither,
10. given an equation in variables x, y, z , identify and sketch the traces of the surface it represents,
11. use the traces to obtain a rough shape of the surface,
12. find the domain and range of a function of two variables,
13. know the definition and geometric significance of level curves,
14. draw the level curves of a function of two variables.