Math 105 - April 10 Final Exam Review

1 Series

a Text: Determine if the following series converge or diverge.

i $\sum_{k=1}^{\infty} \frac{(k!)^3}{(3k)!}$

ii $\sum_{k=2}^{\infty} \frac{5\ln k}{k}$

iii $\sum_{k=1}^{\infty} \frac{2^k k!}{k^k}$

iv $\sum_{k=1}^{\infty} \frac{k^8}{k^{11}+3}$

v $\sum_{k=1}^{\infty} \frac{(k^2+1)^{1/3}}{\sqrt{k^3+2}}$

vi $\sum_{k=1}^{\infty} \frac{1}{3^k-2^k}$

vii $\sum_{k=1}^{\infty} k^{-1/k}$

b Text: Find the values of the parameter p for which $\sum_{k=2}^{\infty} \frac{1}{k\ln k(\ln\ln k)^p}$ converges.

c Text: Evaluate $\sum_{k=1}^{\infty} \frac{1}{(k+1)(k+2)}$

d Text: Find the first four nonzero terms in the Taylor series centred at $x=0$ for $f(x)=\frac{e^x+e^{-x}}{2}$ and determine the radius of convergence.

e Text: Find the function represented by $\sum_{k=1}^{\infty} \frac{(x-2)^k}{3^{2k}}$ and find the interval of convergence.

2 Continuous Probability

a From WebWork (numbers are simplified): A raffle has a grand prize of a Mediterranean cruise valued at \$10,000 with a second prize of a Las Vegas getaway valued at \$2000. If each ticket costs \$10 and 100 tickets are sold, what are the expected winnings for a ticket buyer?

b Past Math 105 Final Exam: A discrete random variable takes only two values, 0 and 1. Find $p=\Pr(X=1)$ if the variance of X is $1/4$.

c Probability Module: Let $p(x)=\frac{1}{\beta}e^{-x/\beta}$ for $0\le x<\infty$ with $\beta>0$. Show this is a density function and compute $\Pr(X\ge 1)$ and $\mathbb{E}(X)$.

d Probability Module: Show that $f(x)=1-e^{-x}$ for $x\ge 0$ and $f(x)=0$ otherwise is a CDF.

e Probability Module: Find the constant k so that $p(x)=k\sin x$, $0\le x\le\pi$ is a PDF.

3 Miscellaneous

a Text: Find the point(s) on the cone $z^2 = x^2 + y^2$ nearest the point $P(1, 4, 0)$.

b Past Math 105 Final Exam: Let $f(x) = \lim_{n\to\infty}[(\frac{1}{3} + \frac{1}{2+(1+\frac{x-1}{n})^3} + \frac{1}{2+(1+\frac{2(x-1)}{n})^3} + ... + \frac{1}{2+(1+\frac{(n-1)(x-1)}{n})^3})]\frac{x-1}{n}$ where $x \geq 0$. Find the equation of the tangent line to $y = f(x)$ at $x = 1$.

c Text: Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 - 2y + 1$ on the set $R = \{(x, y)|x^2 + y^2 \leq 4\}$.

d Past Math 105 Final Exam: $\int \cos(\ln x)dx$.

e Text: $\int \frac{dx}{(81+x^2)^2}$