The University of British Columbia

Midterm 2 - March 15, 2012

Mathematics 105, 2011W T2

Sections 208, 209

Closed book examination Time: 50 minutes

Last Name ____________ **First** ________ **SID** ____________

Instructor names: Djun Kim, Erin Moulding

Special Instructions:

1. A separate formula sheet will be provided. No books, notes, or calculators are allowed. Unless it is otherwise specified, answers may be left in "calculator-ready" form. Simplification of the final answer is worth at most one point.
2. Show all your work. A correct answer without accompanying work will get no credit.
3. If you need more space than the space provided, use the back of the previous page.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

Q	Points	Max
1		60
2		20
3		20
4 (extra credit)		5
Total		100

1. (a) The area function of a curve $y = f(t)$ between 0 and x is given by

$$A(x) = 1 - e^{-\frac{x^2}{2}}.$$

Find all the critical points of f.

(10 points)

(b) Use Simpson's rule to approximate

$$\int_1^5 \frac{dx}{x}$$

with $n = 4$ subintervals. Find a bound on the error. **No need to simplify your answers!**

(5 + 5 = 10 points)

(c) Find the definite integral

$$\int_0^2 \frac{2x}{x^2-1}\,dx.$$

(10 points)

(d) A discrete random variable X takes values 0 and 1 only. If the expected value of X is $\frac{1}{2}$, what is the variance of X?

(10 points)

(e) What is the antiderivative of $\sec^6 x \tan x$?

(10 points)

(f) Solve the initial value problem

$$y' = \frac{\ln x}{x\sqrt{y}}, \qquad y(1) = 4.$$

(10 points)

2. Find the definite integral:

$$\int_0^1 \frac{e^x + 1}{e^{2x} + 3e^x + 2}\, dx.$$

(20 points)

3. During a certain part of the day, the interarrival time (in seconds) between successive phone calls at a central telephone exchange is a continuous random variable X whose probability density function is given by

$$f(x) = \begin{cases} ke^{-kx} & \text{if } x \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

where k is an unknown constant.

(10 + 10 = 20 points)

(a) If the expected value of X is 1/3 seconds, find the value of k.

(b) Find the probability that the time between successive phone calls is more than 2 seconds.

4. (Extra credit) The health officials are studying a flu virus going around a town of 100,000 people. At any given time, a fixed but unknown proportion k of the uninfected individuals gets infected. The people who have caught the virus once develop an immunity, and are not reinfected. At the start of the study, a quarter of the population is already infected. Write down an initial value problem that models the spread of flu in the population. **Do not solve this problem!**

(5 points)

Formula Sheet

You may refer to these formulae if necessary.

Trigonometric formulae:

$$\cos^2 x = \frac{1+\cos(2x)}{2}.$$
$$\sin^2 x = \frac{1-\cos(2x)}{2}.$$

Simpson's rule:

$$S_n = \frac{\Delta x}{3}\Big(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + ... + 4f(x_{n-1}) + f(x_n)\Big).$$
$$E_s = \frac{K(b-a)(\Delta x)^4}{180}, \quad |f^{(4)}(x)| < K \text{ on } [a,b].$$

Indefinite Integrals:

$$\int \sec x\, dx = \ln\big|\sec x + \tan x\big| + C.$$

Probability:

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x)\, dx.$$
$$\mathrm{Var}[X] = \int_{-\infty}^{\infty} (x - \mathbb{E}[X])^2 f(x)\, dx.$$