The University of British Columbia

Midterm 1 - February 3, 2012

Mathematics 105, 2011W T2

Section 207

Closed book examination Time: 50 minutes

Last Name ____________ **First** ____________ **SID** ____________

Instructor name: Michael Lindstrom

Special Instructions:

1. A separate formula sheet will be provided. No books, notes, or calculators are allowed.
2. Show all your work. A correct answer without accompanying work will get no credit.
3. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

Q	Points	Max
1		50
2		20
3		20
4		10
5 (extra credit)		10
Total		100

1. (a) You are given a function $f(x, y)$ with the following properties: $f(1, 2) = 0$; the rate of change of f as x varies (holding y fixed) is 2, and the rate of change of f as y varies (holding x fixed) is -3. Estimate $f(1.2, 1.9)$.

(8 points)

(b) Let $\mathbf{v} = \langle 3, 2, 1 \rangle$ and $\mathbf{w} = \langle 9, -6, 2 \rangle$. Are the two vectors $\mathbf{v}$ and $\mathbf{w}$ parallel, perpendicular or neither? Justify your answer.

(8 points)

(c) Can you find a plane parallel to the (x, y)-plane (i.e., the plane $z = 0$) passing through the point $P(1, 2, -1)$? If yes, find the equation of this plane. If not, explain why not.

(8 points)

(d) A function $f(x)$ is defined on the interval $[-1, 5]$ as follows:

$$f(x) = \begin{cases} \sqrt{1-x^2} & \text{if } -1 \leq x \leq 1, \\ -\sqrt{4-(x-3)^2} & \text{if } 1 \leq x \leq 5. \end{cases}$$

Thus between $[-1, 1]$, the graph of f is above the x-axis and is a semicircle with center $(0, 0)$. Between $[1, 5]$, the graph of f is below the x-axis and is a semicircle with center $(3, 0)$. Compute the value of the integral

$$\int_{-1}^{5} f(x)\, dx.$$

(8 points)

(e) Is there a function $f(x, y)$ such that $\nabla f(x, y) = \langle \sin y - 1, x \cos y - x \rangle$? If not, explain why no such function exists; otherwise find $f(x, y)$. State clearly any result that you use.

(8 points)

(f) Let R be the semicircular region $\{x^2 + y^2 \leq 9,\ x \geq 0\}$. Find the absolute maximum and minimum values of the function

$$f(x, y) = x^2 + y^2 - 4x$$

on the *boundary of the region* R.

(10 points)

2. Find *all* critical points of the function

$$f(x,y) = \frac{x^4}{4} + \frac{y^4}{4} - xy$$

lying in the region $\{(x,y) : y \geq 0\}$. Classify each point as a local maximum, local minimum, or saddle point.

(10 + 10 = 20 points)

3. A paper company makes two kinds of paper, x units of brown and y units of white every month. Given a fixed amount of raw material, x and y must satisfy the the production possibilities curve

$$4x^2 + 25y^2 = 50,000 \quad \text{for } x \geq 0,\ y \geq 0.$$

It costs the company \$23 to produce a single unit of brown paper, and \$42 to produce a unit of white. On the other hand, brown and white paper sell for \$25 per unit and \$52 per unit respectively. Assuming that the company is confident of selling all the units it produces, find using the method of Lagrange mutipliers how many units of brown and white paper it should manufacture every month so as to maximize its total profit.

Clearly state the objective function and the constraint. There is no need to justify that the solution you obtained is the absolute max or min. **A solution that does not use the method of Lagrange multipliers will receive no credit, even if it is correct.**

(20 points)

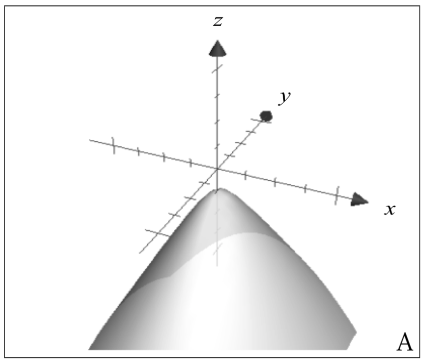
4. Consider the surface S given by
$$z - \frac{x^2}{9} = \frac{y^2}{4}.$$

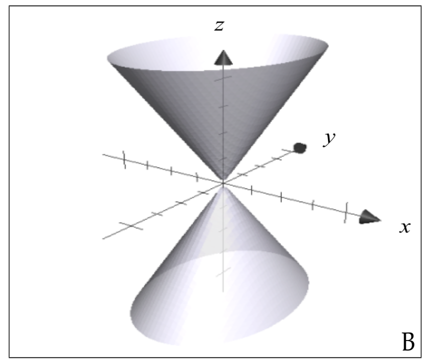
   (a) Sketch the traces of S in the $y = 0$ and $z = 1$ planes, taking care to label the axes and the intercepts of your graph.

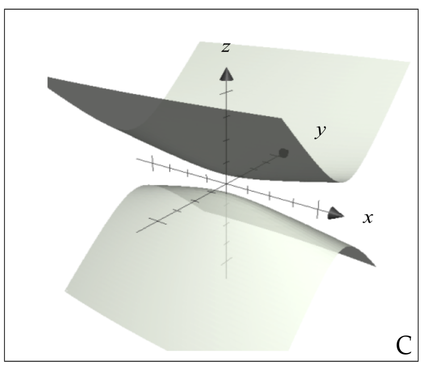
(3 + 3 = 6 points)

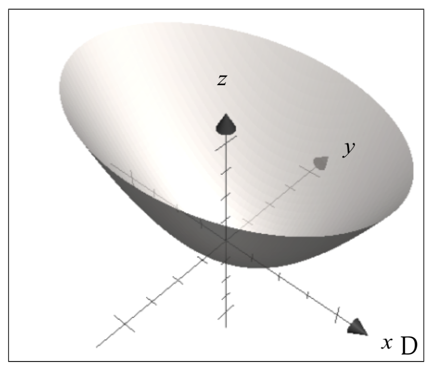
(b) Based on the traces you sketched above, which of the following renderings represents the graph of the surface?

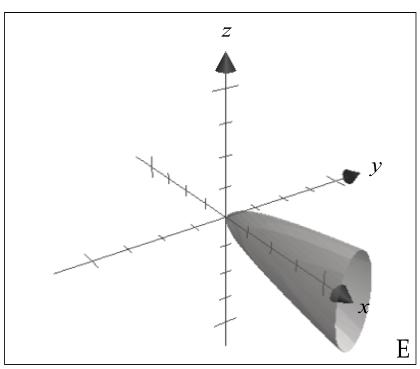
(4 points)

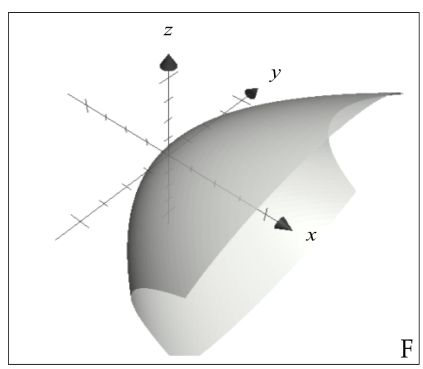












5. (Extra credit) Transform the limit of the following Riemann sum to a definite integral:

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{2}{n}e^{\frac{4k}{n}}.$$

(10 points)

Formula Sheet

You may refer to these formulae if necessary.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$