Math 105 Word Problems 1
Answers

Present Value

1. $\int_0^T K(t)e^{-rt}dt$

2. $\int_0^4 (5,000 - 100t)e^{-0.16t}dt = 14,243$ (use integration by parts)

3. $\int_0^\infty 10000e^{0.04t}e^{-0.12t}dt = 125,000$

4. $\int_0^2 (50 + 7t)e^{-0.1t}dt = 102.9$, so \$102,900.

5. $\int_0^4 50e^{-0.08t}e^{-0.12t}dt = 137.668$, so \$137,668.

Capital Value and Improper Integrals

6. $\int_0^\infty 5000e^{-0,1t}dt = 50,000$

7. $\int_0^\infty Ke^{-rt}dt = \frac{K}{r}$

8. (a) $\sum_{k=1}^{n} M(t_k)\Delta t \approx \int_0^2 M(t)dt$, with $\Delta t = \frac{2}{n}$, $t_k = \frac{2k}{n}$

   (b) $\sum_{k=1}^{n} M(t_k)e^{-0.1t_k}\Delta t \approx \int_0^2 M(t)e^{-0.1t}dt$, with $\Delta t = \frac{2}{n}$, $t_k = \frac{2k}{n}$

9. $80000 + \int_0^\infty 50000e^{-rt}dt \left(= 80000 + \frac{50000}{r}\right)$

Understanding Differential Equations

10. The growth rate is $y' = (.0004)(500)(1000 - 500) = 100$ fish per month.

11. (a) It is decreasing at \$1600 per year.
    (b) The account is earning interest at 4% per year compounded continuously, and money is being withdrawn steadily at the rate of \$2000 per year (or if there are both deposits and withdrawals, the rate of withdrawals is \$2000/year more than the rate of deposits).

12. At the beginning of the broadcast there are 10 people tuned in. The differential equation says that the number of people who have heard the news broadcast after t hours is increasing at a rate that is proportional to the difference between that number and 200,000. So the news spreads quickly initially, but this slows down as the number of people in the know approaches 200,000.

13. $f' = k(C - f), \quad k > 0$

Solving Differential Equations

14. $P' = 0.06P + 2400, \quad P(0) = 1000 \implies P(t) = 41000e^{0.06t} - 40000$

15. $P' = 0.05P - 4800, \quad P(0) = 25000 \implies P(t) = 96000 - 71000e^{0.05t}$

16. $p' = k(1 - p), \quad p(0) = 0 \implies p = 1 - e^{-kt}$

17. $f' = kf, \quad f(0) = 100, \quad f(2) = 115$
We have two initial conditions, but there is also one undetermined constant (k) in the equation, so this will work out.
The general solution is $f(t) = Ce^{kt}$, and plugging in the initial conditions and solving for k and C gives $C = 100$, $k = \frac{1}{2}\ln\left(\frac{115}{100}\right) \approx 0.07$. Therefore $f(t) = 100e^{0.07t}$.
The CCI will reach 200 at approximately $t = 9.9$.

18. $y(p) = \frac{C}{\sqrt{p+3}}$

Areas between curves

19. $\displaystyle\int_2^6 \left(21.3e^{0.07t} - 21.3e^{0.04t}\right) dt \approx 13.0$ billion barrels.

20. The oil consumption rate in 1970 ($t = -4$) was $R(-4) = 21.3e^{0.07 \cdot -4} = 16.1$.
The easiest approach is then to set $s = 0$ in 1970, so that the two rates we want to compare are $16.1e^{0.07s}$ and $16.1e^{0.04s}$, and the difference in total consumption is
$\displaystyle\int_0^4 \left(16.1e^{0.07s} - 16.1e^{0.04s}\right) ds \approx 4.5$ billion barrels.

21. The amount of wood consumed from 1980 to 2000 is $\displaystyle\int_0^{20} 76.2e^{0.03t} dt \approx 2088$ million cubic meters.
About midway through 2003 we have $g(23.5) \approx 0$, so there are no new trees at all. This could be because all the trees are gone.
The excess is $\displaystyle\int_0^{20} \left(76.2e^{0.03t} - (50 - 6.03e^{0.09t})\right) dt$.

22. $(\text{New}P(8)) - (\text{Old}P(8)) = \displaystyle\int_6^8 ((-x^2 + 12x - 20) - (-x^2 + 14x - 24))\, dx = -20$, so it would be less.
