

Math 105 Week 8

February 28, 2011

1. LESSON PLAN

This week we will cover sections 12.4 and 12.5. The underlying theme is differentiation of functions in two variables. Not surprisingly, many of the concepts in these sections bear strong resemblances with the one-variable theory, but there are also new subtleties inherent to the multivariable nature of the questions.

1.1. Section 12.4. This section deals with *partial derivatives*. Given a function f of two variables x and y , the partial derivative of f with respect to x , denoted f_x , will be obtained by treating f as a function of a single variable x (with y frozen), and taking the usual one-variable derivative of f with respect to x . The other partial derivative f_y will be similarly defined, by reversing the roles of x and y .

It is important to understand the geometric meaning of these first-order partial derivatives. For instance $f_x(a, b)$ measures the instantaneous rate of change in f at the point (a, b) as you move in the x -direction along the line $y = b$. Another way of saying the same thing is the following: Suppose you are standing on the surface $z = f(x, y)$ at the point $(a, b, f(a, b))$. As you move on the surface directly above the horizontal line joining (a, b) with $(a + \Delta x, b)$, the height of the surface changes by $f_x(a, b)\Delta x$. Similar interpretations hold for f_y . Such approximations are known as linear approximations. We will return to these issues again in section 12.7.

The functions f_x and f_y in general depend on x and y ; so the above differentiation process may be repeated on these new functions, leading to higher-order partial derivatives. We will mention an interesting result due to Clairaut that says that for nice functions the order in which you take successive partial derivatives does not matter, for example $f_{xy} = f_{yx}$, $f_{xxy} = f_{xyx} = f_{yxx}$.

We will omit the part of section 12.4 on differentiability.

1.2. Section 12.5. Before launching into this section, it may be helpful to review the notion of a curve on a plane and on a surface, parameterized by a scalar t . The main concept in this section is the chain rule in the context of multivariable functions. We will cover the subsections

- chain rule with one independent variable (Theorem 12.7)
- implicit differentiation (Theorem 12.9)

but *will omit the section on chain rule for two independent variables (Theorem 12.8) and hence tree diagrams*. It is important to understand the significance of the chain rule in the context of finding the rate of change of a function along a curve lying on a surface, or equivalently in terms of the slope of a tangent line to a curve lying on a surface.

2. LEARNING OBJECTIVES

By the end of the week and after going through the practice problems,

1. you should be able to compute first and higher-order partial derivatives of a function of several variables.
2. You should know how to interpret first order partial derivatives geometrically in terms of rate of change.
3. You should be comfortable with chain rule involving one independent variable, and its analytic and geometric significance.