

## Math 105 Week 6

February 7th, 2011

### 1. LESSON PLAN

This week we will look at three different topics: a special type of integral called *improper*; a very important application, *differential equations*; and problems involving *areas between curves*. Unlike in previous weeks, these topics will be accompanied by many word problems, showing them in action in (more or less) real-world situations.

**Improper integrals** come in two flavors: with infinite intervals, and with unbounded functions. The first look like for instance  $\int_0^\infty f(x)dx$ , and are defined as a limit of a normal integral on a finite interval. If  $f(x)$  becomes small enough quickly enough as  $x$  goes to  $\infty$ , such an integral can have a finite value – even though it represents the area of an infinitely long region.

**Differential equations** make up an area of mathematics that has found application in all sciences. They show up whenever the rate of change of a function is connected to the value of the function itself, which happens in many applications. Differential equations form a very large field, and we will merely see a small sample, in particular the type of differential equation called *separable*.

**Areas between curves** also show up in many applications. We have already seen these in their simplest form, namely the area between the graph of a function and the  $x$ -axis (aka a definite integral). But now we see more general areas, which will of course be computed using definite integrals, but they require some extra attention.

### 2. LEARNING OBJECTIVES

By the end of the week and after going through the practice problems, you should be able to:

1. distinguish between the different types of improper integrals and explain their definition as limits;
2. evaluate the various kinds of improper integrals;
3. recognize when the solution to a word problem requires an improper integral;
4. explain what a differential equation is;
5. solve separable equations;
6. set up a differential equation for a word problem and solve it;
7. sketch regions between graphs and compute their areas.