Math 105 Week 4
January 24th, 2011

1. Lesson Plan

This week we shall cover Sections 7.3, 7.4 and 7.6 from Chapter 7 of the text book.

In Section 7.3, we will learn an important technique from substitution rule, namely the trigonometric substitution. Using this we shall be able to find anti-derivates of functions that involve fractional powers of $a^2 - x^2$ or $a^2 + x^2$ or $x^2 - a^2$. They will be achieved using a substitution that involve specific trigonometric functions.

In Section 7.4, we learn the "Method of partial fractions". This method (a new one) will helps us find antiderivatives of rational functions. A function f is a rational function if it is given by $f(x) = \frac{p(x)}{q(x)}$ where p and q are polynomial functions. If the degree of p is less than the degree of q then there are many cases but we will consider only two of them. These depend on how q factors. The first case is when q is a product of two linear factors, second case is when q is a product of three linear factors and the last case is when q is an irreducible quadratic factor. If the degree of p is larger than q, a polynomial division (or long division) is first performed and then one proceeds like the former case for the remainder term (if any).

In Section 7.6, we learn a couple of Numerical methods for integration. There are many instances when one cannot find the definite integral exactly (either by using the Fundamental Theorem of Integral Calculus or by Geometry or by a limit of Riemann sums). In such cases one tries to estimate the value of the definite integral. We shall discuss two such rules. One is the midpoint rule (this re-visits the concept of midpoint Riemann sum) and the other is called the Trapezoid rule. In both rules we quantify the absolute and relative error between the estimate and the true value.

2. Learning Objectives

At the end of Section 7.3, one must know:

- using trigonometric substitutions, how to find anti-derivatives of functions of the form $(a^2 - x^2)^p$, $(a^2 + x^2)^p$, $(x^2 - a^2)^p$ when p is a positive or negative fraction and a is a positive number.

- how to calculate the definite integrals for the functions mentioned above and functions that reduce to the above after a u-substitution.

At the end of Section 7.4, one must know how to solve the three cases that occur while trying to find out the anti-derivatives of rational functions. Namely:

- Partial fractions with simple linear factors (two and three)

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- When the integrand is a rational function and the denominator is a simple irreducible quadratic factor.

- Reduction to the above two cases when the degree of numerator is greater than the degree of denominator

At the end of Section 7.6, one must know:

- Midpoint rule

- Trapezoid rule