

Practice problem set 2

1. Find the present value of a deposit whose future value after 8 years at an annual rate of 9%, compounded continuously, is \$15,000.
2. Find the future value of \$4,000 after 10 years at an annual rate of 8%, compounded continuously.
3. Suppose that an amount of money M is deposited in a bank that pays an annual interest rate r , compounded continuously.
 - (a) Find a formula expressing T , the time needed for M to double, as a function of r .
 - (b) Find T if r is 5%.
 - (c) If r is 10%, find the time needed for M to quadruple.
4. Find the interest rate that must be paid by a bank so that an initial deposit doubles in 7 years, assuming that the interest is compounded continuously.
5. Suppose that money is deposited steadily into a savings account at a constant rate of \$20,000 per year. Find the balance at the end of 7 years if the account pays 9% interest, compounded continuously.
6. Suppose that money is deposited steadily into a savings account at a constant rate of \$15,000 per year for 5 years. Find the present value of this income stream if the account pays 7.5% interest, compounded continuously.
7. If a continuous income stream flows into a savings account at a constant rate of \$10,000 per year and earns 8% interest, compounded continuously, find the time required for the balance to become \$2,000,000.
8. When the usual breeding tanks are full, a fish farm uses a lake on its property to store excess breeding fish. If the farm puts fish into the lake at the rate of 1,000 fish per year and if the fish increase their population exponentially at an annual rate of 33%, find the fish population in the lake after 6 years. Assume that no fish dies or is removed.
9. An employee of a company is offered a choice between two retirement plans. In the first plan, the company will deposit an initial amount of \$5,000 in an IRA paying interest at an annual rate of 8%, compounded continuously, and it will then continue to deposit money at a constant annual rate of \$15,000 per year for the next 25 years. In the second plan, the company will pay the employee \$1,100,000 at the end of 25 years. Which one is the more beneficial plan for the employee?
10. A star player in the NBA is offered a 6-year contract by a team and two choices of compensation. In the first he is offered a lump sum of \$40,000,000, paid at the beginning of his contract. In the second he is offered an initial payment of \$6,000,000 and 6-year continuous income stream at the rate of \$7,500,000 per year deposited into a savings account paying 8% annual interest, compounded continuously. Assuming that the player can also invest his money with the same interest of 8%, determine which plan is better for the player.
11. Imagine yourself in the following situation. You have already gotten that great job, and you are thinking of buying that great house 7 years from now. You estimate that you will need

\$100,000 for a down payment and other expenses. Let us also assume that you have an account that pays 6.5% interest, compounded continuously.

(a) If you deposit money continuously into the account at a constant rate during the whole 7-year period, find the rate (dollars per year) you must use in order to meet your goal of \$100,000.

(b) If you decide to make one lump sum deposit now, instead of continuous deposits, what is the amount you must deposit now in order to meet your goal of \$100,000?

12. On the day their daughter is born, a couple begins saving for her college education by depositing money into an account that pays 5.5% interest, compounded continuously. If the deposits are made often enough so that they can be approximated by a continuous income stream flowing at the rate of $5000 + 500t$ dollars per year, calculate the amount that will be in the account on their daughter's eighteenth birthday.

13. Assume that estimates show that a growing company will be generating a continuous income stream for the next 5 years at the variable rate of $25 + 3t$ million dollars per year at any time t .

(a) Compute the future value of the income stream, assuming that money can be invested for that period at the annual rate of 9% interest, compounded continuously.

(b) Compute the present value of the income stream described in (a).

(c) Write an equation relating the numbers found in (a) and (b).

14. The population of a fast-developing region is increasing in two ways. First, its intrinsic growth rate is 2% per year (birth rate minus death rate). Second, its annual net immigration rate (new arrivals minus departures) is given by $S(t) = 0.1t + 0.004$ million per year, where t is the time in years. If the initial population was 4 million, what is the population of the region after 10 years?

15. A self-employed software engineer estimates that her annual income over the next 10 years will steadily increase according to the formula $70,000(1.2)^t$, where t is the time in years. She decides to save 12% of her income in an account paying 6% annual interest, compounded continuously. Treating the savings as a continuous income stream over a 10-year period, find the present value.

16. A 30-year old man plans to invest money as a continuous income stream at a constant rate until he is 70. After that he will stop investing and start withdrawing at an annual rate of \$60,000. If the account pays (and will always pay) 7% interest, compounded continuously, and he wants the fund to pay off for 20 years, at what rate should he invest?

17. The Michigan Fish and Game Commission has built a man-made lake in one of its large state parks, and it plans to stock it with trout. It estimates that fishermen will remove trout from the lake at a steady rate of 6,000 per year. It also estimates that the fish will grow exponentially at a rate of 12% per year. If it does not plan to restock the lake for 5 years, what is the minimum number of trout it must put in the lake now?

18. Suppose you have \$60,000 in a savings account paying 4.5% interest, compounded continuously. If you start withdrawing money continuously at the rate of \$5,000 per year without making any additional deposits, how long will the account last?

Answers

1. 7,301.28 dollars.
2. 8,902.16 dollars.
3. (a) $T = \ln 2/r$.
(b) 13.86 years.
(c) 13.86 years.
4. 10%
5. 195,024.57 dollars.
6. 62,542.14 dollars.
7. 35.42 years.
8. 18,917.
9. The first plan pays 1,234,893.30 dollars and the second one pays 1,100,000 dollars. The first plan is more beneficial for the employee.
10. The first plan pays 40,000,000 dollars and the second one pays 41,739,057.02 dollars. Thus, the second plan is better for the player.
11. (a) 11,281.33 dollars.
(b) 63,444.80 dollars.
12. 269,655.10 dollars.
13. (a) 201.67 million dollars.
(b) 128.59 dollars.
(c) $201.67 \approx 128.59e^{0.09 \cdot 5}$.
14. 10.2806 million.
15. 164,680.82 dollars.
16. 2,926.85 dollars
17. 22,559.
18. 17.26 dollars.