Math 105, Spring 2011

Practice Problems on Present and Future value

1. Suppose that money is deposited daily into a saving account at an annual rate of \$ 1000. If the account pays %5 interest compounded continuously, estimate the balance in the account at the end of 3 years.

Answer: \$3236.68

2. Suppose that money is deposited daily into a saving account at an annual rate of \$ 2000. If the account pays %6 interest compounded continuously, approximately how much will be in the account at the end of 2 years?

Answer: \$4249.90

3. Suppose that money is deposited steadily into a saving account at the rate of \$ 16000 per year. Determine the balance at the end of 4 years if the account pays %8 interest compounded continuously.

Answer: \$75426

4. Suppose that money is deposited steadily into a saving account at the rate of \$ 14000 per year. Determine the balance at the end of 6 years if the account pays %4.5 interest compounded continuously.

Answer: \$96433

5. An investment pays %10 interest compounded continuously. If money is invested steadily at the rate of \$ 5000 per year, how much time is required until the value of investment reaches \$140000?

Answer: $10 \ln 3.8 \approx 13.35$ years

6. A savings account pays %4.25 interest compounded continuously. At what rate per year must money be deposited steadily into account to accumulate a balance of \$100000 after 10 years?

Answer: $\$\frac{4250}{e^{.425}-1} \approx \$8025.07$

7. Suppose that money is to be deposited daily for 5 years into a saving account at an annual rate of \$1000 and the account pays %4 interest compounded continuously. Let the interval from 0 to 5 be divided into daily subintervals of duration $\Delta t = \frac{1}{365}$ years. Let $t_1, \ldots, t_n$ be points chosen from the subintervals.

(a) Show that the present value of a daily deposit at time t_i is $1000\Delta t e^{-.04t_i}$.

(b) Find the Riemann sum corresponding to the sum of the present values of all the deposits.

Answer: Riemann sum $= 1000[e^{-.04t_1} + e^{-.04t_2} + \cdots + e^{-.04t_n}]\Delta t$

(c) What is the function and interval corresponding to the Riemann sum in part (b)?

Answer: $f(t) = 1000e^{-.04t}$ $0 \leq t \leq 5$

(d) Give the definite integral that approximates the Riemann sum in part (b).

Answer: $\int_0^5 1000e^{-.04t}dt$

(e) Evaluate the definite integral in part (d). This number is the *present value of continuous income stream.*

Answer: \$4531.73