Math 105 Assignment 7
Due the week of March 7

1. Compute
$$u_{xx} + u_{yy} \qquad \text{if} \qquad u(x,y) = \ln(x^2 + y^2).$$

(5 points)

2. An ant sits on an overturned bowl shaped like the hemisphere
$$z = \sqrt{1 - x^2 - y^2}.$$
If it starts crawling directly above the curve $x = y = e^{-t}$, $t \geq \frac{1}{2}$ find the rate at which it will be gaining or losing height when it passes the point $(\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}})$. Is it going uphill or downhill at that point?

$$(3+1 = 4 \text{ points})$$

3. Consider the production function of an enterprise

$$f(x, y) = 60x^{\frac{3}{4}}y^{\frac{1}{4}},$$

which gives the numer of units of goods produced when utilizing x units of labor and y units of capital. The enterprise currently employs 81 units of labor and invests 16 units of capital.

(a) If the capital is held fixed at its current value, and the amount of labor increased by one unit, find the approximate change in the quantity of goods produced. This quantity is known as the *marginal productivity of labor*.

(b) Similarly, holding the amount of labor fixed at its current value, determine the approximate increase in productivity generated by an increase in capital by one unit. This is called the *marginal productivity of capital.*

(3+3=6 points)