

Name:

Student number:

Grade: /15

Math 105 Assignment 2

Consider the function $f : [0, 2] \rightarrow \mathbf{R}$ defined by

$$f(x) = \begin{cases} \sqrt{1 - (x - 1)^2}, & 0 \leq x \leq 1, \\ x, & 1 < x \leq 2. \end{cases}$$

(a) Sketch the graph of the function f (2 pts).

(b) Find an antiderivative of $\sqrt{1 - (x - 1)^2}$ (2 pts). **Hint:** use the formula

$$\int \sqrt{1 - x^2} dx = \frac{x\sqrt{1 - x^2}}{2} + \frac{\sin^{-1} x}{2} + C.$$

(c) Calculate the left Riemann sum for a regular partition and $n = 4$. Does the result underestimate or overestimate $\int_0^2 f(x)dx$ (2+1 = 3 pts)?

(d) Calculate the right Riemann sum for a regular partition and $n = 4$. Does the result underestimate or overestimate $\int_0^2 f(x)dx$ (2+1 = 3 pts)?

(e) Evaluate $\int_0^2 f(x)dx$ using a geometric argument. **Hint:** the area of a circle with radius r is πr^2 (2 pts).

(f) Evaluate $\int_0^2 f(x)dx$ using the fundamental theorem of calculus. **Hint:** split the integral into two integrals (3 pts).