Suggested problems from past exams

Math 101, April 2005, Ex. 1.(f).

Evaluate

$$\int_0^\infty \frac{dx}{(x+1)^3}\,.$$

Math 101, April 2005, Ex. 2.(a).

Let R be the finite region bounded above by the curve $y = 4 - x^2$ and below by $y = 2 - x$. Carefully sketch R and find its area explicitly.

Math 101, April 2006, Ex. 1.(i).

Determine whether the integral

$$\int_1^\infty \frac{dx}{1+2x}$$

converges or diverges. If it converges, evaluate it.

Math 101, April 2006, Ex. 2.(a).

Sketch the bounded region that lies between the curves $y = 2 - x^2$ and $y = |x|$. Show that the area of this region equals 7/3.

Math 101, April 2006, Ex. 4.(a).

Solve the initial-value problem

$$\frac{dy}{dx} = y^4(x+1)^2, \;\; y(0) = -1\,.$$

Express your answer in the form $y = f(x)$ and simplify your answer completely.

Math 101, April 2007, Ex. 1.(i).

Evaluate

$$\int_e^\infty \frac{1}{x(\ln x)^2}\,dx\,.$$

Math 101, April 2007, Ex. 2.(a).

Sketch the bounded region that lies between the curves $y = 4 - x^2$ and $y = (x-2)^2$. Find its area.

Math 101, April 2007, Ex. 6.

The population of fish in a lake is m million, where $m = m(t)$ varies with time t (in years). The number of fish is currently 2 million.

(a). Suppose m satisfies the logistic-growth differential equation

$$\frac{dm}{dt} = 16m(1 - \frac{m}{4})\,.$$

When will the number of fish equal to 3 million? You may use the fact that the general solution to the logistic-growth differential equation $y' = ky(1-(y/K))$ is $y = K/(1+Ae^{-kt})$, where A is a constant.

(b). Suppose instead that (because of fishing by humans) m satisfies

$$\frac{dm}{dt} = 16m(1-\frac{m}{4}) - 12\,.$$

Will the fish population ever equal 3 million? You must give justification for your answer.

Math 101, April 2008, Ex. 2(a).

Sketch the bounded region that lies between the curves $y = 2x^2$ and $y = 4 + x^2$. Find its area.

Math 101, April 2008, Ex. 7(a).

Show that the area of the region inside the ellipse $(x/a)^2 + (y/b)^2 = 1$, where a and b are positive real numbers, equals πab.

Math 101, April 2009, Ex. 2.

For what values of p does the following integral converge:

$$\int_e^{\infty} \frac{1}{x(\ln x)^p}\,.$$

Math 101, April 2009, Ex. 6.(a).

Solve the differential equation $y' = xy^2$ with initial condition $y(0) = 1$.

Math 101, April 2009, Ex. 8.

Consider the initial value problem

$$\frac{dx}{dt} = k(0.1-x)(0.2-x) \text{ and } x(0) = 0\,.$$

(a). Solve the initial value problem to find $x(t)$ explicitely.

(b). What value does $x(t)$ approach as t approaches ∞.

Math 101, April 2010, Ex. 1.(f).

Evaluate

$$\int_{-\infty}^{-1} e^{2x}\,dx\,.$$

Math 101, April 2010, Ex. 7.

Find the solution of the differential equation

$$\frac{dL}{dt} = kL^2 \ln t$$

that satisfies $L(1) = 1$. Here k is a constant that will appear in your final answer.

Math 103, April 2005, Ex. 7.

Cholera is a disease that can spread through untreated sewage polluting the drinking water supply. Let $y(t)$ be the fraction of people in a population who have the disease at time t. Assumes that in a certain city, the fraction of people who have the disease increases at a rate proportional to the fraction of people who do not yet have the disease (we will use $k > 0$ as the constant of proportionality).

(a). Write down the differential equation that $y(t)$ satisfies.

(b). Solve this equation, assuming that at time $t = 0$ the fraction of people who have the disease is $y(0) = y_0$, where $0 \leq y_0 \leq 1$.

(c). Use one clear sentence and a simple sketch to explain what happens to the fraction of infected individuals as time goes by.

Math 103, April 2006, Ex. 7.

In an experiment involving a bacteria population, $N(t)$ denotes the size of the population (measured in thousands of individuals) as a function of time starting at $t = 0$. The initial population is $N(0) = N_0$.

(a). Suppose the population growth is governed by the differential equation

$$\frac{dN}{dt} = \frac{N}{t+1}\,.$$

Find $N(t)$ if $N(0) = 2$.

(b). Suppose the population growth is governed by the differential equation

$$\frac{dN}{dt} = \frac{N^2}{(t+1)^2}\,.$$

Find $N(t)$ if $N(0) = 2$.

(c). What happens to the solution from part **(b)** when $t \to 1$? Can you find an initial population N_0 for which this problem doesn't occur for any time t.

Math 103, April 2009, Ex. 3.

Consider the differential equation

$$\frac{dx}{dt} = \frac{x^2 - 2}{2x}$$

(a). Solve the differential equation by separation of variables.

(b). Find the solution with the initial condition $x(0) = 2$

Math 103, April 2010, Ex. 1.5.

Which of the following improper integrals converges?

$$\int_1^\infty e^{3x}dx\,, \quad 10^{-5}\int_1^\infty x^{-1}dx\,, \quad \int_1 x^{-1/2}dx\,, \quad \int_1^\infty x^{-2}dx\,, \quad \int_1^\infty \frac{x^2}{1000}dx\,..$$

Math 103, April 2010, Ex. 1.8.

Consider the differential equation

$$\frac{dy}{dt} = 1 - 4y^2$$

and initial condition $y(0) = 0$. After a long time (when y no longer changes), the value of y is

$$y = 0\,, \quad y = 1\,, \quad y = 1/4\,, \quad y = 4 \quad \text{or} \quad y = 1/2\,.$$

Math 103, April 2010, Ex. 2.

Calculate the area of the bounded region enclosed by the curves $f(x) = x^3 - 4x$ and $g(x) = -x^2 + 2x$ for $x \geq 0$.

Math 103, April 2010, Ex. 6.

The height of fluid $h(t)$ in a cylindical container is controlled by a pump so that it satisfies the differential equation

$$\frac{dh}{dt} = -kh^{1/3}\,, \quad h(0) = h_0\,,$$

where $k > 0, h_0 > 0$ are constants and h_0 is the initial height of the fluid.

(a). Solve this differential equation to determine the height $h(t)$ at any later time t.

(b). At what time will the container be empty?

Math 105, April 2005, Ex. 5

The museum of Calculus is currently 50 meters from the edge of an eroding cliff. The geologists predict that at time t, the distance y, in meters, from the museum to the cliff edge will be changing at a rate given by

$$\frac{dy}{dt} = \frac{-120e^{-0.5t}}{(1 + 4e^{-0.5t})^2}\,.$$

Here t is measured in years, and $t = 0$ is now.

(a). Find a formula for y in terms of t.

(b). Will the distance from the museum to the cliff edge will ever reach 0? Explain.

Math 105, April 2005, Ex. 6

Suppose that $y(0) = 1/5$ and, for all t,

$$\frac{dy}{dt} = y^2 e^{-t}\,.$$

(a). Find an explicit formula for y as a function of t.

(b). Find $\lim_{t\to\infty} y(t)$.

Math 105, April 2006, Ex. 1.

(a). Assume that $z(x, y)$ is a linear function with slope 2 in the x direction and -3 in the y direction. If $z(1, 1) = 4$ find $z(-2, 1)$.

(b). If $f(x,y) = \ln(x^2+y)$, find

$$\lim_{k\to 0} \frac{f(1+k,0)-f(1,0)}{k}.$$

(i). Determine whether the following integral converge or diverge

$$\int_1^\infty \frac{dx}{\sqrt{x}}$$

(k). If k is a non zero constant and $y = -k/t^3$ is a solution of the differential equation $\frac{dy}{dt} = 6t^2y^2$, find k.

Math 105, April 2006, Ex. 2.

Find the total area of all the regions totally enclosed by the graphs of the functions $f(x) = x^3 - 3x + 4$ and $g(x) = x + 4$.

Math 105, April 2006, Ex. 4. Suppose that \$100,000 is deposited in an account paying 5% interest with continuous compounding. Also assume that the money is continually withdrawn from the account at a rate of \$10,000 per year. Find the amount of money in the account at the end of 6 years.

Math 105, April 2006, Ex. 6. Suppose that money is deposited continuously into a savings account at a rate of $200t$ dollars per year for 10 years. No money is withdrawn during the 10 year period. The savings account earns 10% interest, compounded continuously.

(a). Find the amount of money in the account at the end of 10 years.

(b). Find the total amount of interest earned in dollars by the savings account over the 10 year period.

Math 105, April 2009, Ex. 1.

(a). Compute

$$\frac{\partial f}{\partial x}(2,1)$$

if $f(x,y) = e^{(1-x)y}$.

(b). Let $f(x,y) = (2x+y^3)^{10}$. Evaluate

$$\frac{\partial^2 f}{\partial y \partial x}.$$

(c). Find all point(s) where $f(x,y) = x^2 + y^2 + xy + 3x - 7$ may have a relative minimum or maximum.

(j). Find the area under the graph of $y = e^{-2x}$ for $x \geq 0$.

Math 105, April 2009, Ex. 2.

Find the area of the region bounded by $y = 8 - x^2$, $y = -2x$ and $y = -7x$.

Math 105, April 2009, Ex. 5.

A person deposits \$5000 in a bank account and decides to make additional deposits at the rate of B dollars per year, where B is a constant. Suppose that the bank compounds interest continuously at the annual rate of 8% and that the deposits are made continuously into the account.

(a). Set up a differential equation that is satisfied by the amount $f(t)$ in the account at time t.

(b). Determine $f(t)$ (as a function B).

(c). Determine B if the initial deposit is to double in seven years.

Math 105, April 2010, Ex. 1.

(a). Let $f(x,y) = 8x^{1/5}y^{4/5}$. Find $f(2x,2y) - 2f(x,y)$.

(b). Let $f(x,y) = xe^{2y} + y^2$. Evaluate

$$\frac{\partial^2 f}{\partial y^2}\,.$$

(c). Find all point(s) where $f(x,y) = x^3y - 8y - 5x$ may have a relative minimum or maximum.

(k). Suppose that money is deposited steadily into a savings account at the rate of \$3000 dollars per year. Determine the balance at the end of 5 years if the account pays 6% interest compounded continuously.

Math 105, April 2010, Ex. 3.

Find the area of the region bounded by $y = -2/x$, $y = -4\sqrt{x}$ and $y = -x$.

Math 105, April 2010, Ex. 5.

A person purchased a home at the price \$300,000, paid a down payment equal to 20% of the purchased price and financed the remaining balance with a 25 year term mortgage. Assume that the person makes payments continuously at a constant annual rate A and that interest is compounded continuously at the rate of 5%.

(a). Write down the differential equation that is satisfied by the amount of money $y(t)$ owed on the mortgage at time t.

(b). Determine A, the rate of annual payments, that is required to pay off the loan in 25 years.

(c). Determine the total interest paid during the 25 year term mortgage.