Math 421/510, Spring 2008, Homework Set 1
(due on Friday January 18)

Instructions

- Homework will be collected at the end of lecture on Friday.
- You are encouraged to discuss homework problems among yourselves. Also feel free to ask the instructor for hints and clarifications. However the written solutions that you submit should be entirely your own.
- Answers should be clear, legible, and in complete English sentences. If you need to use results other than the ones discussed in class, state the result clearly with either a reference or a self-contained proof.

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1. Recall that a metric space is *separable* if it has a countable, dense subset. Show that $\ell^p(\mathbb{N})$ is separable for $1 \le p < \infty$, but ℓ^∞ is not.
2. We sketched a proof in class of the following result: "All norms in a finite-dimensional normed space are equivalent". Fill in the details of that proof.
3. Show that $L^p(\mu)$ is complete for $1 \le p \le \infty$ and for every positive measure μ.
4. Show that if $1 \le p < \infty$, and I is an infinite set, then $\ell^p(I)$ has a dense set of the same cardinality as I.