

## Homework 2 - Math 440/508, Fall 2011

Due on Monday October 3

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1. From Chapter 2, *Complex Analysis - Stein & Shakarchi*: Exercises 5, 7, 8, 9, 12, 13.
2. Show that the image of any nonconstant entire function is dense in the complex plane.
3. Do there exist functions  $f$  and  $g$  that are analytic at  $z = 0$  that satisfy
  - (a)  $f(1/n) = f(-1/n) = n^{-2}$ .
  - (b)  $g(1/n) = g(-1/n) = n^{-3}$ .
4. Determine all entire functions and satisfy

$$\int_0^{2\pi} |f(re^{i\theta})|^2 d\theta \leq Ar^{2k} \quad \text{for all } 0 < r < \infty,$$

where  $k$  is a positive integer and  $A$  is a positive constant.

5. Identify all polynomials  $P$  such that  $P(z)$  is real if and only if  $z$  is real.
6. Evaluate the following integrals:

(a)

$$\int_0^{2\pi} e^{e^{i\theta}} d\theta.$$

(b)

$$\int_0^{2\pi} e^{e^{i\theta} - i\theta} d\theta.$$

(c)

$$\int_{|z|=1} \frac{|dz|}{|z-a|^2} \quad \text{for } |a| < 1, |dz| = \text{arclength measure.}$$

(d)

$$\frac{1}{2\pi} \int_0^{2\pi} \frac{d\theta}{|ae^{i\theta} - b|^4} \quad \text{for } 0 < a < b.$$