Practice Problem Set

1. The ellipsoid $4x^2 + 2y^2 + z^2 = 16$ intersects the plane y = 2 in an ellipse. Find parametric equations for the tangent line to this ellipse at (1, 2, 2).

(Solution:
$$x = 1 + t, y = 2, z = 2 - 2t$$
)

2. Car A is travelling north on Highway 16 and car B is travelling west on Highway 83. Each car is approaching the intersection of these highways. at a certain moment, car A is 0.3 km from the intersection and travelling at 90 km/h while car B is 0.4 km from the intersection and travelling 80 km/h. How fast is the distance between the cars changing at that moment?

(Solution: -118 km/h)

3. Find the normal direction to the surface given by the equation $x - z = \arctan(yz)$.

(Solution: $\langle 1 + y^2 z^2, z, -(1 + y + y^2 z^2) \rangle$)

- 4. Can there exist a function f(x, y) for which $f_x(x, y) = y + x^2 y$ and $f_y(x, y) = x + xy^2$?
- (Solution: no, by Clairauts' theorem) 5. If $f(x,y) = x(x^2 + y^2)^{-\frac{3}{2}}e^{\sin(x^2y)}$, find $f_x(1,0)$.

(Solution: -2.)

6. Find the equation of the tangent plane and the normal line to the surface xy + yz + zx = 3 at (1, 1, 1).

(Solution: x + y + z = 3, x = y = z)