Review Sheet 2

1. An ellipsoid is created by rotating the ellipse $4x^2 + y^2 = 16$ about the x-axis. Find an equation for the ellipsoid.
2. Find the unit tangent, normal and binormal vectors of the curve
$$\mathbf{r}(t) = \langle \cos t, \sin t, \ln \cos t \rangle$$
at the point $(1, 0, 0)$.
3. Find equations of the normal plane and osculating plane of the curve
$$x = t, \quad y = t^2, \quad z = t^3 \quad \text{at } (1, 1, 1).$$
4. Find the best linear approximation of the function $f(x, y) = \sqrt{y + \cos^2 x}$ at $(0, 0)$.
5. If x, y, z satisfy the relation
$$xyz = \cos(x + y + z)$$
find $\partial z/\partial x$ and $\partial z/\partial y$.
6. Find the dimensions of a rectangular box of largest volume such that the sum of the lengths of its twelve edges is a constant c.
7. Evaluate the integral
$$\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2}\, dz\, dx\, dy.$$
8. Give five other iterated integrals that are equal to
$$\int_{0}^{2} \int_{0}^{y^3} \int_{0}^{y^2} f(x, y, z)\, dz\, dx\, dy.$$
9. Show that there is no vector field $\mathbf{G}$ such that
$$\text{curl } \mathbf{G} = 2x\mathbf{i} + 3yz\mathbf{j} - xz^2\mathbf{k}.$$
10. Evaluate the surface integral
$$\iint_S (x^2 z + y^2 z) dS$$
where S is part of the plane $z = 4 + x + y$ that lies inside the cylinder $x^2 + y^2 = 4$.
11. Use Stokes' theorem to evaluate $\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = x^2 yz\mathbf{i} + yz^2\mathbf{j} + z^3 e^{xy}\mathbf{k}$, S is the part of the sphere $x^2 + y^2 + z^2 = 5$ that lies above the plane $z = 1$ and S is oriented upward.
12. Use the divergence theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$ and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = 0$ and $z = 2$.