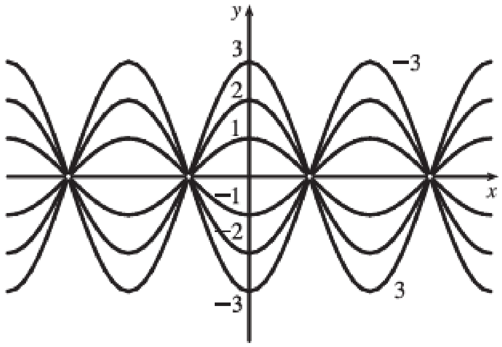
44. $k = y \sec x$ or $y = k \cos x$, $x \neq \frac{\pi}{2} + n\pi$ [n an integer].



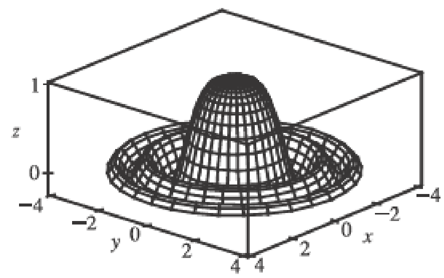
22. $f(x, y, z) = \dfrac{yz}{x^2 + 4y^2 + 9z^2}$. Then $f(x, 0, 0) = 0$ for $x \neq 0$, so as $(x, y, z) \to (0, 0, 0)$ along the x-axis, $f(x, y, z) \to 0$. But $f(0, y, y) = y^2/(13y^2) = \frac{1}{13}$ for $y \neq 0$, so as $(x, y, z) \to (0, 0, 0)$ along the line $z = y$, $x = 0$, $f(x, y, z) \to \frac{1}{13}$. Thus the limit doesn't exist.

38. $f(x, y) = \begin{cases} \dfrac{xy}{x^2 + xy + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$ The first piece of f is a rational function defined everywhere except at the origin, so f is continuous on $\mathbb{R}^2$ except possibly at the origin. $f(x, 0) = 0/x^2 = 0$ for $x \neq 0$, so $f(x, y) \to 0$ as $(x, y) \to (0, 0)$ along the x-axis. But $f(x, x) = x^2/(3x^2) = \frac{1}{3}$ for $x \neq 0$, so $f(x, y) \to \frac{1}{3}$ as $(x, y) \to (0, 0)$ along the line $y = x$. Thus $\lim_{(x,y)\to(0,0)} f(x, y)$ doesn't exist, so f is not continuous at $(0, 0)$ and the largest set on which f is continuous is $\{(x, y) \mid (x, y) \neq (0, 0)\}$.

42. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(x^2 + y^2)}{x^2 + y^2} = \lim_{r\to 0+} \frac{\sin(r^2)}{r^2}$, which is an indeterminate form of type $0/0$. Using l'Hospital's Rule, we get

$$\lim_{r\to 0+} \frac{\sin(r^2)}{r^2} \overset{\text{H}}{=} \lim_{r\to 0+} \frac{2r\cos(r^2)}{2r} = \lim_{r\to 0+} \cos(r^2) = 1.$$

Or: Use the fact that $\displaystyle\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$.



80. (a) $\partial T/\partial x = -60(2x)/(1 + x^2 + y^2)^2$, so at $(2, 1)$, $T_x = -240/(1 + 4 + 1)^2 = -\frac{20}{3}$.

(b) $\partial T/\partial y = -60(2y)/(1 + x^2 + y^2)^2$, so at $(2, 1)$, $T_y = -120/36 = -\frac{10}{3}$. Thus from the point $(2, 1)$ the temperature is decreasing at a rate of $\frac{20}{3}\,^\circ\text{C/m}$ in the x-direction and is decreasing at a rate of $\frac{10}{3}\,^\circ\text{C/m}$ in the y-direction.

6. $z = f(x, y) = e^{x^2 - y^2} \quad\Rightarrow\quad f_x(x, y) = 2xe^{x^2 - y^2}$, $f_y(x, y) = -2ye^{x^2 - y^2}$, so $f_x(1, -1) = 2$, $f_y(1, -1) = 2$. By Equation 2, an equation of the tangent plane is $z - 1 = f_x(1, -1)(x - 1) + f_y(1, -1)[y - (-1)] \quad\Rightarrow$ $z - 1 = 2(x - 1) + 2(y + 1)$ or $z = 2x + 2y + 1$.

40. Let x, y, z and w be the four numbers with $p(x, y, z, w) = xyzw$. Since the largest error due to rounding for each number is 0.05, the maximum error in the calculated product is approximated by $dp = (yzw)(0.05) + (xzw)(0.05) + (xyw)(0.05) + (xyz)(0.05)$. Furthermore, each of the numbers is positive but less than 50, so the product of any three is between 0 and $(50)^3$. Thus $dp \le 4(50)^3(0.05) = 25{,}000$.

42. $\mathbf{r}_1(t) = \langle 2 + 3t, 1 - t^2, 3 - 4t + t^2 \rangle \;\Rightarrow\; \mathbf{r}_1'(t) = \langle 3, -2t, -4 + 2t \rangle$, $\mathbf{r}_2(u) = \langle 1 + u^2, 2u^3 - 1, 2u + 1 \rangle \;\Rightarrow$ $\mathbf{r}_2'(u) = \langle 2u, 6u^2, 2 \rangle$. Both curves pass through P since $\mathbf{r}_1(0) = \mathbf{r}_2(1) = \langle 2, 1, 3 \rangle$, so the tangent vectors $\mathbf{r}_1'(0) = \langle 3, 0, -4 \rangle$ and $\mathbf{r}_2'(1) = \langle 2, 6, 2 \rangle$ are both parallel to the tangent plane to S at P. A normal vector for the tangent plane is $\mathbf{r}_1'(0) \times \mathbf{r}_2'(1) = \langle 3, 0, -4 \rangle \times \langle 2, 6, 2 \rangle = \langle 24, -14, 18 \rangle$, so an equation of the tangent plane is $24(x - 2) - 14(y - 1) + 18(z - 3) = 0$ or $12x - 7y + 9z = 44$.

24. $M = xe^{y-z^2}$, $x = 2uv$, $y = u - v$, $z = u + v \;\Rightarrow$

$$\frac{\partial M}{\partial u} = \frac{\partial M}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial M}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial M}{\partial z}\frac{\partial z}{\partial u} = e^{y-z^2}(2v) + xe^{y-z^2}(1) + x(-2z)e^{y-z^2}(1) = e^{y-z^2}(2v + x - 2xz),$$

$$\frac{\partial M}{\partial v} = \frac{\partial M}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial M}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial M}{\partial z}\frac{\partial z}{\partial v} = e^{y-z^2}(2u) + xe^{y-z^2}(-1) + x(-2z)e^{y-z^2}(1) = e^{y-z^2}(2u - x - 2xz).$$

When $u = 3$, $v = -1$ we have $x = -6$, $y = 4$, and $z = 2$, so $\dfrac{\partial M}{\partial u} = 16$ and $\dfrac{\partial M}{\partial v} = 36$.

44. $f_o = \left(\dfrac{c + v_o}{c - v_s}\right) f_s = \left(\dfrac{332 + 34}{332 - 40}\right) 460 \approx 576.6$ Hz. v_o and v_s are functions of time t, so

$$\frac{df_o}{dt} = \frac{\partial f_o}{\partial v_o}\frac{dv_o}{dt} + \frac{\partial f_o}{\partial v_s}\frac{dv_s}{dt} = \left(\frac{1}{c - v_s}\right) f_s \cdot \frac{dv_o}{dt} + \frac{c + v_o}{(c - v_s)^2} f_s \cdot \frac{dv_s}{dt}$$

$$= \left(\tfrac{1}{332-40}\right)(460)(1.2) + \tfrac{332+34}{(332-40)^2}(460)(1.4) \approx 4.65 \text{ Hz/s}$$