Math 217 Assignment 10
Due Friday December 4

■ Problems to turn in:

1. Evaluate
   (a) the surface integral
   $$\iint_S y dS,$$
   where S is the surface $z = \frac{2}{3}(x^{\frac{3}{2}} + y^{\frac{3}{2}})$, $0 \le x \le 1$, $0 \le y \le 1$.
   (b) the surface integral
   $$\iint_S xz\, dS,$$
   where S is the boundary of the region enclosed by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0$ and $x + y = 5$.
2. Find the flux of $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z^4\mathbf{k}$ across the surface S which is the part of the cone $z = \sqrt{x^2 + y^2}$ beneath the plane $z = 1$ with downward orientation.
3. Evaluate
   $$\int_C (y + \sin x)\, dx + (z^2 + \cos y)\, dy + x^3\, dz$$
   where C is the curve $\mathbf{r}(t) = \langle \sin t, \cos t, \sin 2t \rangle$, $0 \le t \le 2\pi$.
4. Suppose that S is an oriented smooth surface bounded by a simple closed curve C with positive orientation, and f, g have continuous second-order partial derivatives on an open region containing S. Show that
   (a) $\int_C (f\nabla g) \cdot d\mathbf{r} = \iint_S (\nabla f \times \nabla g) \cdot d\mathbf{S}$.
   (b) $\int_C (f\nabla f) \cdot d\mathbf{r} = 0$.
5. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ and
   $$\mathbf{F}(x, y, z) = xy \sin z\mathbf{i} + \cos(xz)\mathbf{j} + y \cos z\mathbf{k}.$$
6. (a) Let $\mathbf{F}(x, y, z) = z \arctan(y^2)\mathbf{i} + z^3 \ln(x^2 + 1)\mathbf{j} + z\mathbf{k}$. Find the flux of $\mathbf{F}$ across the part of the paraboloid $x^2 + y^2 + z = 2$ that lies above the plane $z = 1$ and is oriented upward.
   (b) Evaluate
   $$\iint_S (2x + 2y + z^2) dS$$
   where S is the sphere $x^2 + y^2 + z^2 = 1$.
7. Show that for any smooth vector field in $\mathbb{R}^3$, the flux of the curl of the vector field across any surface enclosing a simple solid is zero.