**Math 263 Assignment 9**
Due November 26

**Problems from the text (do NOT turn in these problems):**
(17.7) 5-13, 19-30. (17.8) 2-10. (17.9) 5-15, 19, 20.

**Problems to turn in:**

1. Find the flux of $\vec{F} = (x^2 + y^2)\vec{k}$ through the disk of radius 3 centred at the origin in the $xy$ plane and oriented upward.

2. For each of these situations, (i) Sketch $S$, (ii) Parametrize $S$, (iii) find the vector and scalar elements $d\vec{S}$ and $dS$ for your parametrization, (iv) calculate the indicated surface or flux integral.

   (a) $S$ given by $z = x^2y^2$, $-1 \leq x \leq 1$, $-1 \leq y \leq 1$ oriented positive up. Calculate $\int\int_S \vec{F}.d\vec{S}$ for $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$.

   (b) $S$ is the surface of $4x^2 + 4y^2 + z^2 - 6z + 5 = 0$ oriented inward. Calculate the surface area of $S$.

   (c) $S$ is the surface of intersection of the sphere $x^2 + y^2 + z^2 \leq 4$ and the plane $z = 1$ oriented away from the origin. Calculate the flux through the surface of the electrical field $\vec{E}(\vec{r}) = \frac{\vec{r}}{|\vec{r}|^3}$.

3. For constants $a, b, c, m$, consider the vector field

$$\vec{F} = (ax + by + 5z)\vec{i} + (x + cz)\vec{j} + (3y + mx)\vec{k}.$$

   (a) Suppose that the flux of $\vec{F}$ through any closed surface is 0. What does this tell you about the value of the constants $a, b, c$ and $m$?

   (b) Suppose instead that the line integral of $\vec{F}$ around any closed curve is 0. What does this tell you about the values of the constants $a, b, c$ and $m$?

4. Let $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$. Consider the vector field

$$\vec{E} = \frac{\vec{r}}{|\vec{r}|^3}.$$

   Find $\int_S \vec{E}.d\vec{A}$ where $S$ is the ellipsoid $x^2 + 2y^2 + 3z^2 = 6$. Give reasons for your calculation.

5. Use geometric reasoning to find $I = \int\int_S \vec{F}.d\vec{S}$ by inspection for the following three situations. Explain your answers. In each case, $a$ and $b$ are positive constants.

   (a) $\vec{F}(x, y, z) = x\vec{i} + y\vec{j} + z\vec{k}$ and $S$ is the surface consisting of three squares with one corner at the origin and positive sides facing the first octant. The squares have sides ($b\vec{i}$ and $b\vec{j}$), ($b\vec{j}$ and $b\vec{k}$), and ($b\vec{i}$ and $b\vec{k}$), respectively.

   (b) $\vec{F}(x, y, z) = (x\vec{i} + y\vec{j})\ln(x^2 + y^2)$, and $S$ is the surface of the cylinder (including top and bottom) where $x^2 + y^2 \leq a^2$ and $0 \leq z \leq b$.

   (c) $\vec{F}(x, y, z) = (x\vec{i} + y\vec{j} + z\vec{k})e^{-(x^2+y^2+z^2)}$, and $S$ is the spherical surface $z^2 + y^2 + z^2 = a^2$.

6. Let $S$ be the boundary surface of the solid given by $0 \leq z \leq \sqrt{4-y^2}$ and $0 \leq x \leq \frac{\pi}{2}$.

   (a) Find the outward unit normal vector field $\vec{N}$ on each of the four sides of $S$.

   (b) Find the total outward flux of $\vec{F} = 4\sin x\vec{i} + z^3\vec{j} + yz^2\vec{k}$ through $S$.

   Do the calculation directly (don't use the Divergence theorem).

7. Evaluate, both by direct integration and by Stokes' Theorem, $\int_C (z\,dx + x\,dy + y\,dz)$ where $C$ is the circle $x+y+z=0$, $x^2+y^2+z^2=1$. Orient $C$ so that its projection on the $xy$–plane is counterclockwise.

8. Evaluate $\int_C (x\sin y^2 - y^2)dx + (x^2 y\cos y^2 + 3x)dy$ where $C$ is the counterclockwise boundary of the trapezoid with vertices $(0,-2)$, $(1,-1)$, $(1,1)$ and $(0,2)$.

9. Evaluate $\int_C \vec{F}.d\vec{r}$ where $\vec{F} = ye^x\vec{i} + (x+e^x)\vec{j} + z^2\vec{k}$ and $C$ is the curve

$$\vec{r}(t) = (1+\cos t)\vec{i} + (1+\sin t)\vec{j} + (1-\sin t-\cos t)\vec{k}$$