Math 263 Assignment 3
Due September 26

Problems from the text (do NOT turn in these problems):
(15.1) 13-18, 23-27, 30-42, 55-60. (15.2) 5-12. (15.3) 5-10, 15-20, 51-56, 70, 74-75. (15.4) 1-6, 11-20, 40, 41, 42. (15.5) 1-12, 21-34, 40.

Problems to turn in:

1. (a) Draw a contour diagram for the function $f(x, y) = \sqrt{(x-1)^2 + (y-2)^2}$. Indicate the contours $f(x, y) = 1, 2, 3$ and 4.
   (b) Calculate $\nabla f(2, 3)$ and indicate this vector on your diagram.
   (c) Consider $z = f(x, y)$. Find the equation of the tangent plane to $f(x, y)$ at the point $(2, 3)$.

2. A function $z = f(x, y)$ is called *harmonic* if it satisfies this equation:
$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.$$
This is called *Laplace's Equation.* Determine whether or not the following functions are harmonic:
   (a) $z = \sqrt{x^2 + y^2}$
   (b) $e^{-x} \sin y$
   (c) $3x^2 y - y^3$

3. In each case, give an example of an appropriate function or show that no such fuction exists.
   (a) A function $f(x, y)$ with continuous second order partial derivatives and which satisfies $\frac{\partial f}{\partial x} = 6xy^2$ and $\frac{\partial f}{\partial y} = 8x^2 y$.
   (b) A function $g(x, y)$ satisfying the equations $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 2xy$.

4. Use the appropriate version of the chain rule to compute the following:
   (a) dw/dt at $t = 3$, where $w = \ln(x^2 + y^2 + z^2), x = \cos t, y = \sin t$, and $z = 4\sqrt{t}$.
   (b) $\partial z/\partial u$ and $\partial z/\partial v$, where $z = xy, x = u \cos v$, and $y = u \sin v$.

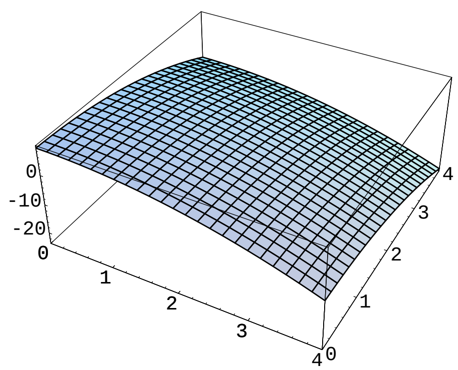
5. Suppose a duck is swimming around in a circle, with position given by $x = \cos t$ and $y = \sin t$. Suppose that the water temperature is given by $T = x^2 e^y - xy^3$. Find the rate of change in temperature that the duck experiences as it passes through the point $(1/\sqrt{2}, -1/\sqrt{2})$.

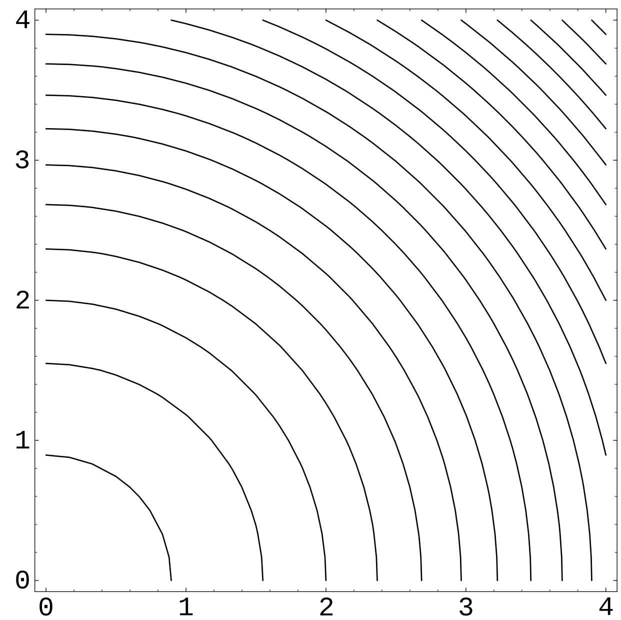
6. Compute the following using implicit differentiation:

   (a) $\partial y/\partial z$ if $e^{yz} - x^2 z \ln y = \pi$.

   (b) dy/dx if $F(x, y, x^2 - y^2) = 0$.

7. The surface plot $z = f(x, y)$ and the contour diagram are shown:





   Look at the point $(2, 2)$. At this point, find the sign (positive or negative) of each of the following quantities:

   - $\partial f/\partial x$
   - $\partial f/\partial y$
   - $\partial^2 f/\partial x^2$
   - $\partial^2 f/\partial y^2$
   - $\partial^2 f/\partial x \partial y$

8. Find the equation of the tangent plane to $z = \sqrt{xy}$ at the point $(1, 1, 1)$.

9. You have three resistors labeled 10Ω, 20Ω and 30Ω. Each of the resistances is guaranteed accurate to within 1%.

   (a) You connect the resistors in series, hoping to get a resistance of 6000Ω. Use differentials to estimate the maximum error in the resistance.

   (b) You connect the resistors in parallel, hoping to get a resistance of $\frac{60}{11}\Omega$. Use differentials to estimate the maximum error in the resistance.