Math 340(921) Problem Set 3

Due in class on Friday 31 May 2013

1. Use the two-phase method to solve this problem:

$$\begin{aligned} \text{maximize} \quad & f = 3x_1 + 2x_2 + 3x_3 \\ \text{subject to} \quad & 2x_1 + x_2 + x_3 \leq 2, \\ & 3x_1 + 4x_2 + 2x_3 \geq 8, \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

2. Study the following problem.

$$\begin{aligned} \text{maximize} \quad & f = 5x_1 + x_2 - x_3 \\ \text{subject to} \quad & 3x_1 + x_2 - x_3 \leq -2 \\ & 3x_1 - x_2 - 2x_3 \leq -3 \\ & x_1 \leq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(a) By comparing the objective function and the constraints, or otherwise, show that this problem cannot be unbounded.

(b) Find a basic feasible solution.

(c) Find all optimal solutions.

3. Suppose D^0 is a feasible simplex dictionary in which index E is eligible to enter the basis and index L is eligible to leave. After making this pivot, the next dictionary is named D^+, and its objective row will have the form

$$f = v^+ + \sum_{j \in \mathcal{N}^+} c_j^+ x_j.$$

(a) Write out the objective row in dictionary D^+, using coefficients from dictionary D^0.

(b) Deduce that index L is not eligible to [re-]enter the basis in dictionary D^+.

[The handout entitled "The Simplex Method in Complete Detail" provides convenient notation for this task.]

4. The Ministry of Highways requires a mixture of sand and salt to spread on BC's roads during the winter. The mixture must contain not more than 70% sand and not less than 10% salt. (It may also contain gravel, dirt, and other materials.) Products available include Carol's Road Mix, with 75% sand and 2% salt for \$50/ton, and Gord's Grits, with 60% sand and 6% salt for \$120/ton. Pure salt costs \$800/ton. What combination of the two mixtures and salt meets the requirements with the lowest cost per ton?

5. Sam plans to sell apartment-cooling fans next April, May, and June. He can get up to 450 fans per month from the supplier, and his customers will buy up to 600 fans per month. Sam plans to

More . . .

increase the price of a fan as the weather warms up, and his supplier knows this, so the supplier will do the same. This table shows the prices for a single fan.

Month	Wholesale Purchase Price	Retail Selling Price
Apr	\$31	\$40
May	\$33	\$44
Jun	\$36	\$48

Sam can store up to 300 fans from April until May, for a storage cost of \$2 per fan. The same rules applies to fans stored from May until June, but at the end of June, all the fans must be sold.

Help Sam maximize his profit by giving him detailed advice on how many units to buy, sell, and hold in each month. What profit do you predict?

6. Consider this nonstandard linear program:

$$(P): \quad \left[\begin{array}{ll} \text{minimize} & f = 150 - 6x_1 + 2x_2 - 9x_3 \\ \text{subject to} & 2x_1 - 6x_2 - \ x_3 \le 10 \\ & x_1 + \ x_2 + 9x_3 \le 20 \\ & 0 \le x_1 \le 5,\ x_2 \ge -6,\ x_3 \ge 1 \end{array}\right.$$

(a) Write a standard linear program "equivalent" to (P).

(b) Solve the standard linear program in (a), using computer assistance if you like.

(c) Report the optimum value for (P) and a point (x_1, x_2, x_3) that achieves it.

7. A florist makes bouquets of roses and carnations in these three popular styles:

Code Name	Carnations	Roses	Selling Price
1. Friendship	5	2	\$5.50
2. Romance	3	6	\$10.50
3. Forgiveness	12	4	\$13.00

The flowers come from local wholesalers, who can supply up to 85 dozen (that's $85 \times 12 = 1020$) carnations for \$5.40/dozen and up to 75 dozen roses for \$14.40/dozen. There is another carnation wholesaler in Surrey, who can provide up to 65 dozen at a price of \$9.00/dozen. The florist can sell all the bouquets she makes, and wants to maximize her profit.

(a) Let x_1, x_2, and x_3 denote the number of arrangements of each type; let x_4 denote the number of carnations imported from Surrey. Present a standard-form linear programming problem that captures the story above.
Hint: The florist's total expenditure on carnations can be found by charging the base rate for each one actually used, and then adding a surcharge for each one that was imported.

(b) Use suitable computer software to solve the problem you wrote in (a). (Do not insist on integer answers.)

(c) Translate the computer's solution into clear policy recommendations for the florist: how many dozen flowers should be ordered from each source, and how many of each bouquet should then be made?

(d) What would your recommendations be if the grower in Surrey dropped the price of carnations to \$6.60 per dozen? (All other considerations remain unchanged.)