Math 340(921) Problem Set 2

Due in class on Friday 24 May 2013

1. (a) Use the simplex method to solve the following problem:

$$\begin{array}{ll} \text{maximize} & f = 4x_1 + 2x_2 + 2x_3 \\ \text{subject to} & x_1 + 3x_2 - 2x_3 \le 3, \\ & 4x_1 + 2x_2 \le 4, \\ & x_1 + x_2 + x_3 \le 2, \\ & x_1, x_2, x_3 \ge 0. \end{array}$$

Use Anstee's pivot-selection rules; report the maximum value and the point that attains it.

(b) Suppose the objective function f in the part (a) is replaced with

$$g = 3x_1 + 2x_2 + x_3.$$

Adjust the optimal final dictionary from part (a) to produce a feasible dictionary for the g-problem. Then answer these questions. Is the maximizing point from part (a) still a maximizer for g? If so, explain why; if not, find the new maximum value and all points that achieve it.

(c) [Much like (b).] Suppose the objective function f in part (a) is replaced with

$$h = 4x_1 + 4x_2.$$

Adjust the optimal final dictionary from part (a) to produce a feasible dictionary for the h-problem. Is the maximizing point from part (a) still a maximizer for h? If so, explain why; if not, find the new maximum value and all points that achieve it.

Discussion: Parts (b)–(c) give you personal experience of re-using work you have already done to save time when solving a new problem that is pretty similar to a known one.

2. Use the Simplex Method with dictionaries to solve textbook problem 2.1(a), page 26. Show all your work, then write a summary giving the optimal value, the optimal solution, and the sequence of feasible basic solutions that the Simplex Method visits on its way to the maximizing point.

3. Read textbook problem 1.6 about the meat packing plant. The story in the question setup is translated into a standard-form LP that appears on page 465. Look at that, too. Then ...

(a) Explain the meaning of each variable in the given LP formulation. Show that your explanation predicts a net profit of $9965 for the sample schedule in the question statement.

(b) Use a computer to solve the given LP. Find the maximum profit (it should be larger than $9965!) and present a table showing the schedule that achieves it. Also hand in a computer printout showing exactly what you typed into the solver and what it returned.

NOTES: Use any computer package you like—just say which one you choose. Possibilities:

1. Use LINDO, a special linear programming solver available on the machines in the Math/Stat undergraduate computer lab, room LSK 121. Access instructions should have arrived in

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your email last week. No special skills are required: just start LINDO, select "Help" from the menu, and try things.

2. Use the online simplex method tool cited on the course web page. The tool has an "example" button that shows you how it works, and another sample is posted on the course web page.

Most of the work in part (b) is in figuring out how to communicate your mathematical problem to the software, and how to interpret the results. These are skills worth learning now: once you have them, you can check your manual calculations and/or solve larger problems with ease.

4. Consider the following problem, in which a real parameter k appears:

$$\begin{array}{lrl} \text{maximize} & f = -3x_1 + x_2 + kx_3 & \\ \text{subject to} & x_1 - x_2 & \leq 0, \\ & -2x_1 + x_3 & \leq 1, \\ & -2x_2 + x_3 & \leq 2, \\ & x_1 + x_2 - x_3 & \leq 6, \\ & x_1, x_2, x_3 & \geq 0. \end{array}$$

Notice that changing the value of k makes no difference to the feasible region.

(a) Suppose $k = 1$. Apply the Simplex Method and answer these questions:

(i) Is the problem bounded?

(ii) If the problem is bounded, find all maximizing points and their corresponding values.

(iii) If the problem is unbounded, find a feasible point whose f-value is larger than 10^{100}.

(b) Repeat part (a) with $k = 2$.

5. Consider a general feasible dictionary.

$$x_{n+i} = b_i - \sum_{j=1}^{n} a_{ij} x_j, \qquad i = 1, 2, \ldots, m$$

$$z = v + \sum_{j=1}^{n} c_j x_j$$

in which $x_1, \ldots x_n$ are non-basic and $x_{n+1}, \ldots, x_{n+m}$ are basic. Prove that if we can choose an entering variable, but there are no leaving variables then the objective function can be made arbitrarily large and so the problem is unbounded.

6. Find all maximizing points (if any) and their objective value:

$$\begin{array}{lrl} \text{maximize} & f = 3x_1 + x_2 & \\ \text{subject to} & x_1 + 2x_2 & \leq 5, \\ & x_1 + x_2 - x_3 & \leq 2, \\ & 7x_1 + 3x_2 - 5x_3 & \leq 20, \\ & x_1, x_2, x_3 & \geq 0. \end{array}$$

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7. Consider this problem involving a real parameter k:

$$\begin{array}{lrcl} \text{Maximize } f = & x_1 + x_2 + kx_3 & & \\ \text{Subject to} & x_1 & \leq & 2 \\ & x_2 & \leq & 2 \\ & x_3 & \leq & 4 \\ & 4x_1 + 4x_2 + x_3 & \leq & 16 \\ & x_1, x_2, x_3 \geq 0 & & \end{array}$$

(a) Use the simplex method to solve the problem when $k = -1$. Identify all maximizing points.

(b) Find all k-values for which this problem has a unique maximizing point. Identify the maximizer.

(c) Solve the problem when $k = 1$. Identify all maximizing points.

(d) Find all k for which the set of maximizers in the problem is a line segment in (x_1, x_2, x_3)-space. Identify the endpoints of the segment.

(e) Find all k (if any) for which the set of maximizers includes 3 or more different basic feasible solutions. Identify those points.