2013 Calculus Examination — Comments

[42] **1.** (a) L'Hospital's Rule works here, of course, but factoring the difference of squares is more elementary and direct.

(b) Sign errors or forgetting "$+C$" cost a few students a mark here.

(c) Generally OK, but watch for bracketing issues in the chain rule!

(d) Straightforward.

(e) Most got the right derivative form. However, many decided to have more than one value for a, which was $a = -2$. They plugged an equation of b (in terms of a) into the derivative to set up a quadratic equation for a. If they successfully rejected the spurious root, full marks were given.

(f) Very few students earned any marks here. There was an alarming tendency to assert that $a = b$ because the differences were negligible. (Indeed, the differences were small by design—so that the question would require something beyond simplistic reliance on a calculator.) Of the few who succeeded, most compared a^2 with b^2; the link to linear approximation and concavity was recognized by only one candidate!

(g) No particular comments.

(h) Since only the value of y' at the point of interest is required, there is no need to construct a general formula for y' before plugging in $(x, y) = (1, 2)$.

(i) No particular comments.

(j) No particular comments.

(k) No particular comments.

(l) No particular comments.

(m) Only a few succeeded here. Some were got the right answer using L'Hospital's rule. Those who substituted $f(x) = 5x$, presumably because it is one of the many functions for which $f'(3) = 5$, received only partial credit: an important part of the task here is showing that the result is $-3/5$ for **all** f with the given property, not just one.

(n) Various approaches exist here. The key point is that $\sqrt{x^2}$ equals $|x|$, not simply x, and that this makes a difference when $x < 0$.

[10] **2.** Far too many students struggled to write formulas for the volume of a cylinder, the perimeter of a circle, and/or the area of a rectangle. Those who were able to set up the function $A = A(r)$ generally fared quite well. Of the various approaches to showing that $r = \sqrt[3]{V/\pi}$ is a true minimizer, analysis of the sign of $A'(r)$ on both sides of the critical point is by far the best. Noting that $A''(r)$ has a positive value right at the critical point was accepted, though it actually provides less information.

Simple numerical comparisons between $A(4)$ and the adjacent values from $A(3)$ and $A(5)$, however, were deemed too crude to merit the single point reserved for this justification.

[10] **3.** Here again, translating the geometrical situation into equations was the principal obstacle. The rate of change for the volume of water in the upper chamber was often done well, but then the widespread tendency to launch into calculations without precisely defining variables generated a lot of difficulty in the lower chamber. Many students used $h_2 = 10$ instead of the correct value, $h_2 = 8$.

[10] **4.** Almost everybody got part (a), and part (b) went well, too—presumably because students had been explicitly told that $v(t) = 0$ when $t = 2$. Part (c) looks like just a small step beyond part (b), but it was too much for most students—even though the ideas involved are just the same.

[10] **5.** (a) Almost everyone found y' correctly, but still a few got the tangent line equation wrong and then didn't find a correctly. (Most of these errors were problems with signs.)

(b) Only a very small number of writers succeeded. Most students wrote Newton's root-finding formula correctly, but almost all of them applied it to the function $h(x) = x^3 + 1$ instead of to the true function of interest.

[12] **6.** Many students were able to use the given graph to determine (at least approximately) the intervals where $g(x) = f'(x)$ is negative and positive, or where g is decreasing or increasing. These intervals tell when f is decreasing or increasing, or concave-down or concave-up, respectively. The idea of area under the graph of $g = f'$ revealing changes in the values of f, however, was picked up by only a handful of writers. (The given fact that $f(-1) = 0$ provides an "anchor" or "base point" for the area estimates of interest.) area estimates of

[6] **7.** (a) A strong majority of students named the Intermediate Value Theorem, but mistakenly applied it to $f(x) = x + \ln|x|$ on the interval $[-1, 1]$. Most of these writers even knew that the IVT applies only to continuous functions—so they just said that f is continuous on the given interval, when in fact it has an essential discontinuity at 0.