2010 Calculus Challenge Exam - Comments

1. Many students had difficulty with one or more parts for this question.
   (a) The most common error was not recognizing that $x-2$ and $2-x$ are opposites.
   (b) The most common error was not recognizing that $|x+4| = -(x+4)$ for $x < -4$.
   (c) Overall this part was done poorly. Most students tried to apply L'Hospital's Rule which does not work for this problem. (Applying L'Hospital's Rule twice yields the same limit that we started with.)

2. Overall this question was done very well. There were some problems with notation, such as writing the derivative as a differential or expressing the derivative as $\frac{dy}{dx}y'$.

3. On average this question was done well. About one in ten students ignored the instruction in boldface to use the definition of the derivative. Some students that wrote the correct limit did not know how to rationalize the numerator.

4. This question was done very well. Some students did not give the general antiderivative, omitting the constant of integration. Other errors were treating e^x as a power function and writing the antiderivative of $\cos x$ as $-\sin x$.

5. Overall this question was done very well. A few students made calculation errors and did not appear to check whether their answers seemed reasonable when they calculated the area to be negative.

6. (a) Many students struggled to explain why the equation has at least one solution.
   (b) This part was mostly well done. Some students did not attempt it, claiming that they had not seen Newton's method.

7. This question had very mixed results, ranging from very well done to very poorly done. Some students had contradicting answers from part to part. Common errors included ignoring the instruction to use limits to identify any asymptotes, expressing the asymptotes correctly, and substituting $\pm\infty$ when taking limits as $x \to \pm\infty$.

8. (a) This part was well done. Some students over-complicated the problem by trying to find an antiderivative of g' but did not manage. A few claimed that they had not seen linear approximations.

   (b) This part was also well done. Most students calculated g'' correctly, as suggested in the hint, but some did not know how to use it to address the question asked. A few students confused concave up with concave down.

9. (a) This part was very well done.

   (b) Only about one in ten students solved this part correctly. Many confused velocity and speed.

   (c) About half of the students solved this part correctly. Some students over-complicated the problem by approximating areas under the graph of the velocity function, rather than using the graph of the position function which they identified correctly in part (a).

10. Overall this question was done well.

    (a) Implicit differentiation was done correctly for the most part. However, many students used $y = \sqrt{x^3 + 5x^2}$ without any justification for why they did not consider $y = -\sqrt{x^3 + 5x^2}$.

    (b) This part was done very well.

    (c) Some students did not provide any justification for their answer for this part.

11. The majority of students did well on this question. The most common error was to use the wrong sign in the model.

12. This question was mostly well done. The most common error was omitting the negative sign for the rate at which the length of rope is changing. Some students presented their solutions poorly and did not define any variables that they introduced.

13. Many students struggled with this question and had difficulty expressing the cross-sectional area as a function of θ. Among the students that set up the problem correctly, many neglected to check for a maximum value at the endpoints of the domain of the function.

14. This problem was challenging for many students, yet about one in four managed to solve it correctly. The most common errors were finding the general antiderivative and identifying the point of tangency.